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Discrete Optimization

γ-Robust facility relocation problem

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ABSTRACT

In this paper, we consider relocating facilities, where we have demand changes in the network. Relocations are performed by closing some of the existing facilities from low demand areas and opening new ones in newly emerging areas. However, the actual changes of demand are not known in advance. Therefore, different scenarios with known probabilities are used to capture such demand changes. We develop a mixed integer programming model for facility relocation that minimizes the expected weighted distance while making sure that relative regret for each scenario is no greater than γ. We analyzed the problem structure and developed a Lagrangian Decomposition Algorithm (LDA) to expedite the solution process. Numerical experiments are carried out to show the performance of LDA against the exact solution method.

1. Introduction and literature review

Facility location problems have been widely studied by many researchers on a variety of sectors. Examples include public facilities, supply chain facilities, healthcare facilities and humanitarian relief facilities that are located to best serve communities. These facilities are often intended to serve the communities for long durations, where we can expect demand changes or shifts in the communities.

Suppose we already have a set of facilities in place and the customer demand has changed over time. Because of these demand changes, existing facilities may no longer be able to provide adequate service, which may yield to an intolerable increase in total weighted distance traveled by the customers (Durukan-Sonmez and Lim, 2012). Therefore, we need to consider relocating some of the existing facilities to new locations that would better serve the customers. In the cases where we have an accurate and reliable forecast for the demand, those values can be utilized to make the optimal relocation decision. However, there could be many instances where obtaining exact figures of the new demand can be challenging because of the difficulties in predicting mobility as well as in and out migration (Gregg et al., 1988). Note that, demand at a point depends on many factors such as community growth and economic vitality (Serra and Marianov, 1998), or the demand itself varies within different time periods. For such instances, it is more reasonable to treat demand as an uncertain parameter.

Demand uncertainty is usually modeled in two different ways (Owen and S Daskin, 1998). The first approach assumes possible values for the demand with probabilities associated with those values. The second approach considers upper and lower bound values for the demand. In both cases, the demand can be represented with various scenarios. Those type of problems are usually handled using robust optimization approaches, i.e., minimizing the maximum regret or worst case objective function (Snyder, 2006). But, a drawback of robust optimization is that worst-case scenario dominates the outcome, even though it may be less likely to occur in reality. This could be a good approach for location of facilities that deals with emergency management situations such as nuclear reactors or ambulance stations. For other types of public or private facilities, decisions made by considering the worst case scenario may be too pessimistic because it is possible that few extreme scenarios, which are less likely to occur, could heavily influence the results of min–max based robust opimizaion. Such decisions may lead to unnecessarily higher expected weighted distances. In order to overcome this issue of the traditional robust optimization techniques, different approaches were suggested in the literature. Daskin et al. (1998) proposed an α-reliable minimax regret model for a *p*-median problem. They minimized the maximum regret of the total weighted distance over a set of scenarios whose total probability is at least α. Chen et al. (2006) introduced an α-reliable mean-excess regret model in which the expected regret was minimized with respect to the scenarios whose total probability of occurrence is no more than $(1-\alpha)$. Snyder and Daskin (2006) proposed a model that minimizes the expected cost while having a relative regret in each scenario no more than a desired amount. This approach provides a balance and trade-off between robustness and expected travel distance/cost.

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Many papers deal with location of facilities under demand uncertainty (Mirchandani and Odoni, 1979; Weaver and Church, 1983; Serra and Marianov, 1998; Conde, 2007) whereas only a few considered relocations for such environments. Carson and Batta (1990) define a stochastic network in which demand at each node varies throughout the day. Due to this variation, one static location for the facility may increase the system-wide response time. In order to minimize the average response time, they considered relocation of the facility throughout the day. In their problem, facility relocation is perceived as an option to react to the changes in demand. Although, relocation for each scenario can be acceptable for mobile facilities such as ambulances, for other type of facilities such as libraries, schools, bank branches or ATMs, it is not reasonable to consider relocating facilities for every scenario. Furthermore, they did not consider costs for relocation in the model.

For non-mobile facilities, the approach proposed by Gregg et al. (1988) could be utilized in order to find facility relocations. Assuming a probability distribution for the demand, they model a public facility system as a network with the objective of minimizing the sum of operating cost for regular and over time, travel distance, overage cost and underage cost. In their approach, they extensively used sensitivity analysis such as finding the weights in the objective function, and opening and closing different combinations of facilities to observe the impact of these decisions. They run the model with all facilities open and obtain the utilization percent for each facility. Based on the utilization, they decide to open and close some of the facilities then rerun the model, and repeat this procedure until a satisfactory solution was found. A major drawback is that the facility opening and closing decisions are exogenous. A better approach could determine which facilities to open and close within an optimization model.

Therefore, the goal of this paper is to provide a new solution approach for the facility relocation problem under demand uncertainty. We develop a mathematical model and a solution algorithm to find a relocation decision that performs well under all scenarios rather than finding different alternatives for each scenario. The optimization model and solution algorithm determine which facilities to open and close while balancing the expected and worst case performance of the decisions. This is achieved by the objective of minimizing the expected weighted distance and the constraint on restricting the relative regret of each scenario not to be greater than γ. The rest of this paper is organized as follows. Section 2 describes the methodology, where we develop the mathematical model. In Section 3, we discuss the solution algorithm that we propose to solve the problem. In Section 4, we introduce the numerical results to show effectiveness of our model and the proposed algorithm. We conclude this paper in Section 5 with future research directions.

2. Methodology

We present a formulation for γ-robust facility relocation problem, γ-RFRP in short. Our goal is to minimize the expected weighted distance while making sure that relative regret for each scenario is less than γ. The relative regret associated with a scenario is calculated as the difference between the total weighted distance corresponding to a location decision and the optimal total weighted distance under that scenario divided by the optimal total weighted distance of the scenario. An optimal weighted distance for each scenario is obtained by solving the deterministic facility relocation problem introduced by Wang et al. (2003), which will be referred to as *dFRP* in the rest of the paper. The following notations and input parameters are used to formulate *dFRP* and γ-RFRP.

$\mathbb{V}_1$	Set of existing facilities
$\mathbb{V}_2$	Set of potential facilities
$\mathbb{V}$	Set of all locations, $\mathbb{V} = \{\mathbb{V}_1 \cup \mathbb{V}_2\}$
v_i	Location $i \in \mathbb{V}$ in the network
$\mathbb{S}$	Set of all demand scenarios
w_{ik}	Demand of customer at $v_i \in \mathbb{V}$ at scenario $k \in \mathbb{S}$
d_{ij}	Distance between customer at $v_i \in \mathbb{V}$ and facility at $v_j \in \mathbb{V}$
p	Number of initial facilities
q	Number of final facilities
o_j	Opening cost of facility at $v_j \in \mathbb{V}_2$
c_j	Closing cost of facility at $v_j \in \mathbb{V}_1$
b	Available budget for relocations
β_k	Probability of scenario $k \in \mathbb{S}$
ζ_k^*	Optimal objective function value of *dFRP* for scenario $k \in \mathbb{S}$
γ	Maximum value of relative regret permitted for each scenario

2.1. Problem formulation for dFRP

We have two sets of decision variables for *dFRP*.

$$y_j = \begin{cases} 1, & \text{if facility at } v_j, j \in \mathbb{V} \text{ is open} \\ 0, & \text{otherwise.} \end{cases}$$

$$x_{ijk} = \begin{cases} 1, & \text{if demand at } v_i, i \in \mathbb{V} \text{ is assigned to facility at } v_j, j \in \mathbb{V}, \\ & \text{in scenario } k \in \mathbb{S} \\ 0, & \text{otherwise.} \end{cases}$$

Then the problem formulation for *dFRP* for a given scenario k is

$$P1: \quad \min \zeta_k = \sum_{i\in\mathbb{V}}\sum_{i\in\mathbb{V}} w_{ik} d_{ij} x_{ijk} \tag{2.1}$$

$$\text{s.t.} \sum_{j\in\mathbb{V}_1} c_j(1-y_j) + \sum_{j\in\mathbb{V}_2} o_j y_j \leqslant b, \tag{2.2}$$

$$\sum_{j\in\mathbb{V}} y_j = q, \tag{2.3}$$

$$\sum_{j\in\mathbb{V}} x_{ijk} = 1, \quad \forall i \in \mathbb{V}, \tag{2.4}$$

$$x_{ijk} \leqslant y_j, \quad \forall i,j \in \mathbb{V}, \tag{2.5}$$

$$y_j, x_{ijk} \in \{0,1\}, \quad \forall i,j \in \mathbb{V}. \tag{2.6}$$

The objective of the formulation is to minimize the total weighted distance. Constraint (2.2) is the budget constraint for opening and closing facilities. Constraint (2.3) makes sure that exactly q facilities are located. Constraint (2.4) states that demand at each location v_i is assigned to a facility. Constraint (2.5) makes sure that demand at location i can be satisfied by facility at v_j only if the facility is open.

2.2. Problem formulation for γ-RFRP

Using the same decision variables (y_j and x_{ijk}) defined in the previous section, γ-RFRP is formulated as follows.

$$P2: \quad \min \sum_{k\in\mathbb{S}}\sum_{i\in\mathbb{V}}\sum_{j\in\mathbb{V}} \beta_k w_{ik} d_{ij} x_{ijk} \tag{2.7}$$

$$\text{s.t.} \sum_{i\in\mathbb{V}}\sum_{j\in\mathbb{V}} w_{ik} d_{ij} x_{ijk} \leqslant (1+\gamma)\zeta_k^*, \quad \forall k \in \mathbb{S} \tag{2.8}$$

$$\sum_{j\in\mathbb{V}_1} c_j(1-y_j) + \sum_{j\in\mathbb{V}_2} o_j y_j \leqslant b \tag{2.9}$$

$$\sum_{j\in\mathbb{V}} y_j = q \tag{2.10}$$

$$\sum_{j\in\mathbb{V}} x_{ijk} = 1, \quad \forall i \in \mathbb{V}, \quad \forall k \in \mathbb{S} \tag{2.11}$$

$$x_{ijk} \leqslant y_j, \quad \forall i,j \in \mathbb{V}, \quad \forall k \in \mathbb{S} \tag{2.12}$$

$$y_j, x_{ijk} \in \{0,1\}, \quad \forall i,j \in \mathbb{V}, \quad \forall k \in \mathbb{S} \tag{2.13}$$

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The objective of the formulation is to minimize the expected weighted distance. Constraint (2.8) makes sure that relative regret for each scenario is no more than γ. The remaining constraints are similar to the ones defined in $P1$.

It is proven that *dFRP* is *NP-hard* (Wang et al., 2003). Since we need to solve a *dFRP* for each scenario in γ-RFRP, it is easy to see that our problem is also *NP-hard*. Note that the problem size of γ-RFRP rapidly grows when we increase the number of facilities as well as future scenarios. We have empirically verified that the computational time increases with the growth of the problem size. Therefore, we propose Lagrangian Decomposition Algorithm to solve our problem in a timely manner.

3. Lagrangian Decomposition Algorithm for γ-RFRP

Lagrangian Decomposition Algorithm (LDA), which is also known as Variable Splitting Algorithm provides equal or better lower bounds than Lagrangian relaxation (Guignard and Kim, 1984; Barcelo et al., 1991; Snyder and Daskin, 2006). LDA allows separation of variables by introducing a new set of variables that are made to be equal to the existing variables in the model. Then, two sub-problems are obtained by relaxing this equality constraint. In our application, we utilize Lagrangian relaxation by adding the equality constraint and one of the complicated constraints to the objective function by multiplying with Lagrangian coefficients. Then, we utilize this solution to generate an upper bound and use a subgradient algorithm to optimize the multipliers. Details of all these procedures are explained in the following sections.

In order to apply LDA, we modify our model by adding a new set of binary variables τ_{ijk} that should be equal to x_{ijk} by constraint (3.7) in the following formulation, P3. The objective functions of P2 and P3 are made to be equal by using the parameter σ, $0 \leqslant \sigma \leqslant 1$.

$$P3: \quad \min \sigma \sum_{i\in\mathbb{V}}\sum_{j\in\mathbb{V}}\sum_{k\in\mathbb{S}} \beta_k w_{ik} d_{ij} x_{ijk} + (1-\sigma)\sum_{i\in\mathbb{V}}\sum_{j\in\mathbb{V}}\sum_{k\in\mathbb{S}} \beta_k w_{ik} d_{ij} \tau_{ijk} \tag{3.1}$$

$$\text{s.t.} \sum_{i\in\mathbb{V}}\sum_{j\in\mathbb{V}} w_{ik} d_{ij} x_{ijk} \leqslant (1+\gamma)\zeta_k^*, \quad \forall k \in \mathbb{S} \tag{3.2}$$

$$\sum_{j\in\mathbb{V}_1} c_j(1-y_j) + \sum_{j\in\mathbb{V}_2} o_j y_j \leqslant b \tag{3.3}$$

$$\sum_{j\in\mathbb{V}} y_j = q \tag{3.4}$$

$$\sum_{j\in\mathbb{V}} x_{ijk} = 1, \quad \forall i \in \mathbb{V}, \quad \forall k \in \mathbb{S} \tag{3.5}$$

$$x_{ijk} \leqslant y_j, \quad \forall i,j \in \mathbb{V}, \quad \forall k \in \mathbb{S} \tag{3.6}$$

$$x_{ijk} = \tau_{ijk}, \quad \forall i,j \in \mathbb{V}, \quad \forall k \in \mathbb{S} \tag{3.7}$$

$$y_j, x_{ijk}, \tau_{ijk} \in \{0,1\}, \quad \forall i,j \in \mathbb{V}, \quad \forall k \in \mathbb{S} \tag{3.8}$$

3.1. Lower bound generation

We obtain a lower bound for the γ-RFRP, by adding the constraints (3.3) and (3.7) to the objective function by multiplying with Lagrangian coefficients u and l, respectively. The optimal solution for the relaxed problem provides a lower bound for P3. Moreover, relaxing those constraints allows to decompose P3 into two subproblems. Solving these subproblems separately is easier than solving P3 itself and the sum of the objective function values of the subproblems provides a lower bound for P3. The two subproblems are demonstrated in the following formulations:

SubProblem 1:

$$\min \sum_{i\in\mathbb{V}}\sum_{j\in\mathbb{V}}\sum_{k\in\mathbb{S}} \sigma \beta_k w_{ik} d_{ijk} x_{ijk} + u\left[\sum_{j\in\mathbb{V}_1} c_j(1-y_j) + \sum_{j\in\mathbb{V}_2} o_j y_j - b\right] - \sum_{i\in\mathbb{V}}\sum_{j\in\mathbb{V}}\sum_{k\in\mathbb{S}} l_{ijk} x_{ijk}$$
$$\text{s.t.} \sum_{j\in\mathbb{V}} y_j = q$$
$$x_{ijk} \leqslant y_j, \quad \forall i,j \in \mathbb{V}, \quad \forall k \in \mathbb{S}$$
$$y_j, x_{ijk} \in \{0,1\}, \quad \forall i,j \in \mathbb{V}, \quad \forall k \in \mathbb{S}.$$

SubProblem 2:

$$\min \sum_{i\in\mathbb{V}}\sum_{j\in\mathbb{V}}\sum_{k\in\mathbb{S}} (1-\sigma)\beta_k w_{ik} d_{ijk} \tau_{ijk} + \sum_{i\in\mathbb{V}}\sum_{j\in\mathbb{V}}\sum_{k\in\mathbb{S}} l_{ijk} \tau_{ijk}$$
$$\text{s.t.} \sum_{i\in\mathbb{V}}\sum_{j\in\mathbb{V}} w_{ik} d_{ij} \tau_{ijk} \leqslant (1+\gamma)\zeta_k^*, \quad \forall k \in \mathbb{S}$$
$$\sum_{j\in\mathbb{V}} \tau_{ijk} = 1, \quad \forall i \in \mathbb{V}, \quad \forall k \in \mathbb{S}$$
$$\tau_{ijk} \in \{0,1\}, \quad \forall i,j \in \mathbb{V}, \quad \forall k \in \mathbb{S}.$$

In order to solve the first subproblem, we reorganize its objective function as follows:

$$\min \sum_{i\in\mathbb{V}}\sum_{j\in\mathbb{V}}\sum_{k\in\mathbb{S}} (\sigma \beta_k w_{ik} d_{ijk} - l_{ijk}) x_{ijk} - \sum_{j\in\mathbb{V}_1} u c_j y_j + \sum_{j\in\mathbb{V}_2} u o_j y_j + \sum_{j\in\mathbb{V}_1} u c_j - bu \tag{3.9}$$

For each v_j, the contribution of opening a facility at v_j to the objective function can be denoted as

$$\rho_j(u,l) = \begin{cases} \sum_{k\in\mathbb{S}}\sum_{j\in\mathbb{V}} \min\{0, (\sigma\beta_k w_{ik} d_{ijk} - l_{ijk})\} - u c_j, & \text{if } j \in \mathbb{V}_1, \\ \sum_{k\in\mathbb{S}}\sum_{j\in\mathbb{V}} \min\{0, (\sigma\beta_k w_{ik} d_{ijk} - l_{ijk})\} + u o_j, & \text{if } j \in \mathbb{V}_2 \end{cases}$$

Since the last two terms of Eq. (3.9) are constant, we rank the ρ_j's in ascending order and we set $y_j = 1$ for each of the q smallest ρ_j to find the optimal solution for the first subproblem. Consequently, solution for x_{ijk} can be obtained as follows:

$$x_{ijk} = \begin{cases} y_j, & \text{if } \sigma\beta_k w_{ik} d_{ijk} - l_{ijk} < 0, \\ 0, & \text{otherwise} \end{cases}$$

The second subproblem can be divided into $|\mathbb{S}|$ instances, and for each instance $k \in \mathbb{S}$:

$$\min \sum_{i\in\mathbb{V}}\sum_{j\in\mathbb{V}}\sum_{k\in\mathbb{S}} ((1-\sigma)\beta_k w_{ik} d_{ijk} + l_{ijk}) \tau_{ijk}$$
$$\text{s.t.} \sum_{i\in\mathbb{V}}\sum_{j\in\mathbb{V}} w_{ik} d_{ij} \tau_{ijk} \leqslant (1+\gamma)\zeta_k^*$$
$$\sum_{j\in\mathbb{V}} \tau_{ijk} = 1, \quad \forall i \in \mathbb{V}$$
$$\tau_{ijk} \in \{0,1\}, \quad \forall i,j \in \mathbb{V}.$$

Each instance is similar to 0–1 Multiple Choice Knapsack Problem (MCKP). In 0–1 MCKP, we need to select exactly one item from multiple disjoint subsets. The goal is to maximize (minimize) the objective function while satisfying the $\leqslant(\geqslant)$ knapsack constraints (Martello and Toth, 1990). In the second subproblem of our decomposition, the assignment of facilities to each customer $i \in \mathbb{V}$ can be considered as a subset. Objective function coefficient and constraint coefficient for each facility j in each subset i is $((1-\sigma)\beta_k w_{ik} d_{ijk} + l_{ijk})$ and $w_{ik} d_{ij}$, respectively.

We know that 0–1 MCKP is *NP-hard* (Martello and Toth, 1990), and using exact solution techniques for the second subproblem would be too time consuming, especially for larger instances. As our goal in solving the second subproblem is to obtain a lower bound for the original problem, the second subproblem does not need to be solved optimally to obtain that lower bound. We used a linear programming based algorithm (Sinha and Zoltners, 1979) to solve the second subproblem. Two important but easy transformations are performed to apply their solution technique because the algorithm requires nonnegative objective function coefficients and a greater than or equal to ($\geqslant$) sign in the knapsack constraint (Snyder, 2003).

$$
\begin{aligned}
P4:\ \min & \sum_{i=1}^{m}\sum_{j\in N_i} c_{ij}x_{ij} \\
\text{s.t.} & \sum_{i=1}^{m}\sum_{j\in N_i} a_{ij}x_{ij} \leqslant b \\
& \sum_{j\in N_i} x_{ij} = 1, \quad k = 1,\ldots,m \\
& x_{ij} \in \{0,1\}, \quad j \in N_i, \quad i = 1,\ldots,m
\end{aligned}
$$

Suppose P4 is a simplified version of each instance k of SubProblem 2, where $c_{ij} = ((1-\sigma)\beta_k w_{ik} d_{ijk} + l_{ijk})$ and $a_{ij} = w_{ik} d_{ij}$ for $\forall k \in \mathbb{S}$. Since the algorithm requires ($\geqslant$) constraint, we first calculate $\bar{a} = \max\{b/m, \max_{j\in N_i, i=1,\ldots,m} a_{ij}\}$. Then, we set $a_{ij} = \bar{a} - a_{ij}$ and $b = m\bar{a} - b$. In addition, negative objective function values may incur while applying the subgradient algorithm (Section 3.3). Therefore, in order to ensure nonnegative coefficients we make a simple adjustment to coefficients, calculate $\bar{c} = |\min\{0, \min_{j\in N_i, i=1,\ldots,m} c_{ij}\}|$ and add $\bar{c}$ to each c_{ij}. After solving the problem, we subtract $m\bar{c}$ from the objective function value.

3.2. Upper bound generation

An upper bound can be obtained from the solution of the first subproblem. We set $y_j = 1$ for the facilities that are decided to remain open in the optimal solution of the first subproblem, then we assign each customer to its closest facility. We first check if the solution is feasible with respect to the budget constraint. If it is feasible, we calculate the relative regret for each scenario and check if all regrets are smaller than or equal to γ. If so, we can say that the solution is feasible with respect to the robustness constraint and it provides an upper bound for the original problem.

If the solution is not feasible with respect to the robustness constraint, we apply a local neighborhood search (LNS) to obtain a local optimal solution. In LNS, we attempt to swap each facility with one of its closest f vertices. We first check if the swap satisfies the budget constraint. Then, we check if the solution after the swap satisfies the γ-robustness constraint by calculating the new relative regrets. If any of the swaps satisfy both constraints, the solution after the swap can be used as an upper bound for the original problem.

An initial and hypothetical upper bound for the algorithm can be obtained using the following proposition:

Proposition 1. $\sum_{k\in\mathbb{S}} \beta_k(1+\gamma)\zeta_k^*$ provides an upper bound for γ-RFRP.

Proof. Let ζ_k^* be the optimal objective function value for each scenario and based on the problem definition, each scenario can have a relative regret of at most γ. Therefore, the total travel distance in any scenario of γ-RFRP can be at most $(1+\gamma)\zeta_k^*$ and the objective function value of γ-RFRP is bounded above by $\sum_{k\in\mathbb{S}} \beta_k(1+\gamma)\zeta_k^*$. □

3.3. Subgradient algorithm for lagrangian multipliers

In order to solve the decomposed problem, we use a subgradient algorithm to calculate the Lagrangian multipliers. The following notations are used for the procedure.

$\bar{z}^*$	Best upper bound
$\underline{z}^*$	Best lower bound
δ^t	Dual gap in each iteration
η, θ_{ijk}	Subgradients of the Lagrangian multiplier u and l_{ijk}
π_1, π_2	Step size coefficients for the Lagrangian multiplier u and l_{ijk}
μ_1, μ_2	Step sizes for the Lagrangian multiplier u and l_{ijk}
t	Iteration index

The algorithm terminates when either one of the following two stopping criteria is met:

1. dual gap (δ) is less than equal to a pre-determined threshold value, or
2. when the maximum number of iterations (t_{max}) has been reached.

The initial value of the Lagrangian multiplier u is set to 0. For the multiplier l_{ijk}, we determine the closest f vertices for each customer $i \in \mathbb{V}$ and for each scenario $k \in \mathbb{S}$, assign the average demand of all customers multiplied by a closeness coefficient.

Step 0: Initialize the parameters
Set $t = 0$, $z_l^* = -\infty$, $\bar{z}^* = \sum_{k\in\mathbb{S}} \beta_k(1+\gamma)\zeta_k^*$, and $\delta^0 = \infty$,
Let $u^0 = 0$, and

$$
l_{ijk}^0 = \begin{cases} \bar{w}_{ik}\frac{f+2-\rho}{f+1}, & \text{if facility at } v_j \text{ is the } \rho \text{ th closest facility to customer } i, \\ & 1 \leqslant \rho \leqslant f, \\ 0, & \text{otherwise} \end{cases}
$$

while $(\delta^t > \delta) \| (t \leqslant t_{max})$
{
Step 1: Solve subProblems 1 and 2 and obtain a lower bound, $\underline{z}^t$, by adding the objective function values of both problems.
If $\underline{z}^t \geqslant \underline{z}^*$, set $\underline{z}^* = \underline{z}^t$.
Step 2:
if y_j satisfies the budget constraint **then**
  assign each customer to the closest facility
  if the current solution satisfies the robustness constraint **then**
    Calculate $\bar{z}^t$
  else
    Perform LNS to find a feasible solution and calculate the $\bar{z}^t$
  end if
    if $\bar{z}^t \leqslant \bar{z}^*$, **then** set $\bar{z}^* = \bar{z}^t$
else
  Go to *Step 1*
end if
Step 3: Calculate the dual gap: $\delta^t = (\bar{z}^* - \underline{z}^*)/\underline{z}^*$
Step 4: Calculate η, θ and step sizes and update the Lagrangian multipliers using the following equations:

$$
\begin{aligned}
\eta &= \sum_{j\in\mathbb{V}1} c_j(1-y_j) + \sum_{j\in\mathbb{V}1} o_j y_j - b \\
\theta_{ijk} &= -x_{ijk} + \tau_{ijk} \\
\mu_1^t &= \pi_1(\bar{z}^* - \underline{z}^*)/\eta^2 \\
\mu_2^t &= \pi_2(\bar{z}^* - \underline{z}^*) \Big/ \sum_{i\in\mathbb{V}}\sum_{j\in\mathbb{V}}\sum_{k\in\mathbb{S}} \theta_{ijk}^2 \\
u^{t+1} &= \max\{0, u^t + \eta\mu_1^t\} \\
l_{ijk}^{t+1} &= l_{ijk}^t + \theta_{ijk}\mu_2^t
\end{aligned}
$$

}

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4. Numerical results

In this section, we present our numerical results to test the γ-RFRP model discussed in sections Sections 2 and 3. All numerical results presented in this section were obtained on a Pentium 4 Xeon 3.6 gigahertz workstation with 4 gigabytes RAM.

4.1. Experiment setup

The γ-RFRP was tested on 25 randomly generated networks with 100 and 250 nodes in each network for the comparison of exact method and proposed LDA. For large scale problems, the LDA was tested on 25 randomly generated networks with $n = 500$. Opening and closing costs were randomly generated from uniform distributions over [200, 300] and [50, 100], respectively. The budget scenarios for opening and closing facilities were set to 1000, 1500, and 3000. The number of initial facilities was set to 4 and locations for these facilities were randomly determined. The total number of final facilities, q, was set to 8. In this setup ($p = 4$, $q = 8$, max $\{o_j\} = 100$, and max $\{c_j\} = 300$), assigning 3000 to b is equivalent to a budget constraint without the limit. If we wish to close all existing 4 facilities and open brand new 8 facilities, the maximum required budget (2800) would be still less than 3000.

Different demand scenarios were generated using an approach similar to the one discussed in Daskin et al. (1998). In each scenario, we created more intense demand in some areas of the network. For this purpose, we define some locations for each scenario which are named attraction points. Locations that are closer to an attraction point have a higher demand than the rest. Demand of each v_i in a scenario k is calculated using Eq. (4.1).

$$w_{ik} = w_i^0 + W_{total}\left(\frac{1/d_{ik}}{\sum_{j\in\mathbb{V}} 1/d_{ij}}\right). \qquad (4.1)$$

The initial demand for vertices, w_i^0, which are used as an input for Eq. (4.1) are generated randomly from a uniform distribution over [100, 200]. Parameter W_{total} is the total of w_i^0's in the network, i.e. $W_{total} = \sum_{i\in\mathbb{V}} w_i^0$. The parameter d_{ik} is the distance between v_i and the attraction point defined for scenario k.

Attraction points are located in the following regions of the network: southeast, northeast, southwest, northwest, center, south, north, west, and east. For example, in the first scenario we have more intense demand in the southeastern part of the network and in the eighth scenario, we have more intense demand in the western part of the network. The probabilities of scenarios for $\mathbb{S}, |\mathbb{S}| = 9$ are as follows:

$$\beta = [0.01, 0.04, 0.15, 0.02, 0.34, 0.14, 0.09, 0.16, 0.05].$$

For LDA, values of some parameters were determined after trial-and-error. The value of σ was set to 0.2. Initial values for Lagrangian multiplier coefficients π_1 and π_2 were set to 1.5 and 2, respectively. Both coefficients were decreased by 10% at every 30th unimproved iteration.

4.2. Experiment results

4.2.1. Exact Solution Approach vs. Lagrangian Decomposition Algorithm

In this section we present the results of our experiments that compare the exact solution approach and LDA. The exact solution approach, which is the binary integer programming model, was coded in GAMS (Brook et al., 2009) and solved by CPLEX 12.1. The LDA was coded in C++.

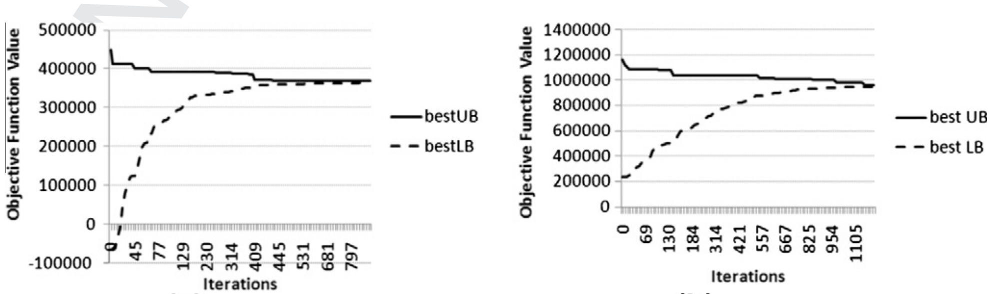
For each instance, we first solved the *dFRP* corresponding to each scenario $k \in \mathbb{S}$, to acquire input parameter ζ_k, $\forall k \in \mathbb{S}$. Then, we solved each instance with both the exact method and the LDA, and recorded the objective function values and the solution time in CPU seconds. Both methods were stopped if the objective function value was within a given dual gap, i.e. 3% and 5% in our experiments.

Fig. 1a and b illustrate the convergence of the LDA for two instances for $n = 100$ and $n = 250$, respectively. In LDA, the lower bound increases rapidly in the initial iterations and the increase becomes slower after some point. On the other hand, the upper bound slowly decreases over the iterations. In both instances, these figures show that the dual gap eventually converged to 1.4% and 0.9%, respectively.

The γ-RFRP has constraints on maximum allowable relative regret (γ) for each scenario and budget (b) for relocations. Due to these limitations on γ and b, some instances may not have feasible solutions (Snyder, 2006). In Lagrangian decomposition algorithm, if a lower bound calculated at any iteration has a value greater then the theoretical upper bound that was found using Proposition 1, then the problem is identified as infeasible. In such instances, no feasible solution can be obtained or either γ or b values should be increased to obtain feasible solutions. Table 1 shows the number of feasible instances out of 25 instances we created for $n = 100$ and $n = 250$ for $b = 1000$, 1500 and 3000 as well as $\gamma = 0.1$, 0.15, 0.2 and 0.25.

Tables 2 and 3 show the average actual dual gap and desired dual gaps for both the exact solution method and LDA for $n = 100$ and 250, respectively. The average solution time and percent time gain of LDA over the exact method for each case are also compared. The percent time gain is calculated by subtracting the average solution time of the LDA from the exact method and dividing it by the solution time of the exact method.

As we can observe from Tables 2 and 3, objective function values for both methods are within the desired dual gap. Solution time gain in its average CPU time of LDA over the exact approach for $n = 100$ ranges from 15% to 94%. A substantial time gain, more than 84% is observed for all cases with $n = 250$.



(a) $n = 100$ **(b)** $n = 250$

Fig. 1. Convergence of the LDA.

Table 1
Number of feasible instances.

Network size	Budget	Gamma			
		0.25	0.2	0.15	0.1
100	1000	25	19	8	1
	1500	25	21	15	3
	3000	25	25	21	5
250	1000	21	21	20	6
	1500	24	24	24	15
	3000	25	24	24	17

4.2.2. Large scale experiments

In this section, we present the numerical results for larger scale problems. As we mentioned in the experiment setup, these experiments include 25 networks each having 500 nodes. We solved each instance using both methods. We stopped the algorithms after 1 hour. The exact method could not find an integer feasible solution at the end of an hour for any of the instances. In fact, no integer feasible solutions were found for several hours of run. Therefore, we could not make a comparison between two methods for large scale problems. We report the average dual gap obtained using LDA in Table 4.

Since some of the instances were infeasible, the numbers in parentheses indicate the number of feasible instances that are used to calculate the average value. As we can see from the numbers, infeasibility increases when we have smaller γ values and less budget available for relocations. Table 4 shows that LDA can generate good quality solutions within the given time limit for large scale problems.

Table 4
Dual gap using LDA for $n = 500$.

Gamma	Budget		
	1000	1500	3000
0.25	2.9% (25)	2.7% (25)	2.3% (25)
0.2	2.9% (24)	2.8% (24)	2.3% (25)
0.15	2.9% (21)	2.6% (22)	2.1% (24)
0.1	3.0% (11)	2.9% (12)	2.5% (16)

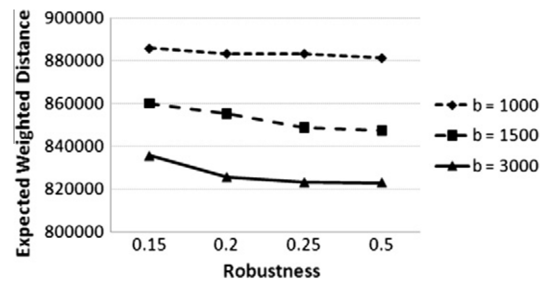


Fig. 2. Expected weighted distance vs. robustness w.r.t. budget.

Table 2
Comparison of LDA and exact solution method for $n = 100$.

Budget	γ	$\delta = 5\%$					$\delta = 3\%$				
		Dual gap		Solution time			Dual gap		Solution time		
		Exact (%)	LD (%)	Exact	LD	Gain (%)	Exact (%)	LD (%)	Exact	LD	Gain (%)
1000	0.25	1.6	4.4	75	42	44	0.9	2.7	103	55	47
	0.2	2.2	4.3	77	48	38	1.7	2.7	87	66	24
	0.15	2.0	4.1	114	62	46	1.7	2.6	117	71	39
	0.1	0.5	2.4	147	32	78	0.5	2.4	147	38	74
1500	0.25	2.2	4.3	261	55	79	1.6	2.7	277	65	77
	0.2	2.8	4.1	814	65	92	2.0	2.5	862	87	90
	0.15	1.8	3.9	871	77	91	1.6	2.3	930	102	90
	0.1	1.2	4.2	1676	104	94	1.2	2.4	1677	118	93
3000	0.25	0.3	4.2	59	47	20	0.2	2.5	60	51	15
	0.2	0.9	3.7	108	59	45	0.6	2.5	109	64	41
	0.15	1.0	4.0	205	62	70	1.0	2.5	213	68	68
	0.1	0.4	4.1	229	59	74	0.4	2.4	229	63	72

Table 3
Comparison of LDA and Exact Solution Method for $n = 250$.

Budget	γ	$\delta = 5\%$					$\delta = 3\%$				
		Dual Gap		Solution Time			Dual Gap		Solution Time		
		Exact (%)	LD (%)	Exact	LD	Gain (%)	Exact (%)	LD (%)	Exact	LD	Gain (%)
1000	0.25	1.1	4.6	3127	229	93	0.9	2.8	3420	301	91
	0.2	1.1	4.6	3582	241	93	0.9	2.8	4034	320	92
	0.15	1.1	4.5	7639	282	96	1.0	2.8	8329	403	95
	0.1	0.9	4.6	30602	279	99	0.9	2.9	30602	470	98
1500	0.25	1.2	4.1	3362	296	91	0.8	2.5	3709	340	91
	0.2	0.9	3.9	2977	316	89	0.8	2.5	3549	349	90
	0.15	1.2	3.9	6825	345	95	1.0	2.5	7040	418	94
	0.1	0.9	4.3	3147	444	86	0.7	2.6	3322	547	84
3000	0.25	0.1	3.1	3353	354	89	0.1	2.0	3353	365	89
	0.2	0.3	3.3	6789	358	95	0.3	2.2	6789	371	95
	0.15	0.1	3.2	3082	407	87	0.1	2.2	3082	428	86
	0.1	0.4	3.2	6250	417	93	0.4	2.2	6250	431	93

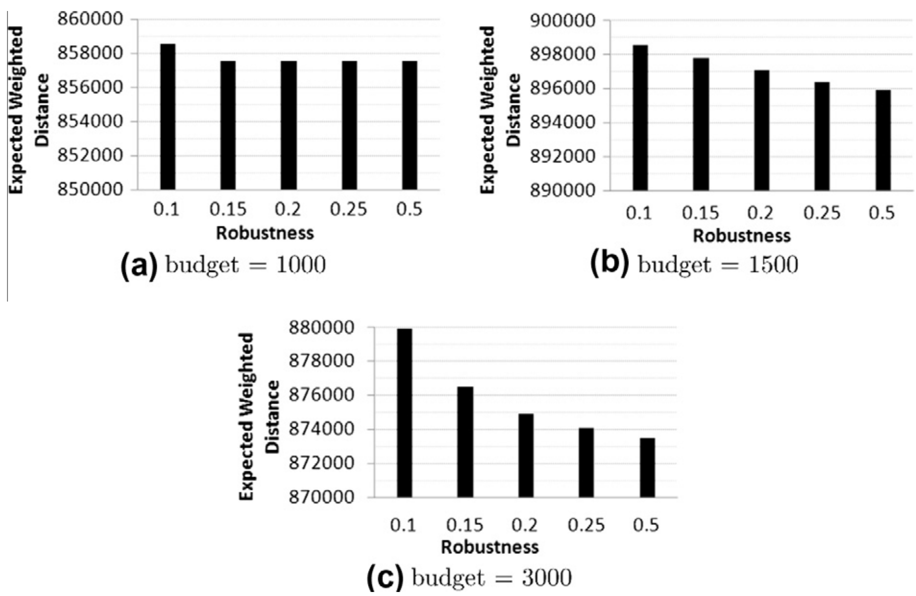


Fig. 3. Expected weighted distance vs. Robustness.

4.2.3. Objective function value vs. γ and budget values

In γ-RFRP problem, both the maximum relative regret permitted for each scenario, which is γ, and the available budget for relocations is used as a constraint in the formulation where the objective is to minimize the expected weighted travel distance from each demand node to its closest facility. Therefore, any increase in those parameters is expected to decrease the objective function value. On the other hand, any decrease in those parameters may increase the objective function value or yield infeasible solution space.

In this section, we analyze the trade off between the objective function value and the parameters γ and budget values. The parameter values used for these experiments are $n = 250$, $\gamma = 0.15, 0.2, 0.25, 0.5$ and *budget* = 1000, 1500, 3000.

Fig. 2 shows the change in the expected weighted distance with respect to the γ values for each budget level. Average objective function values of 20 feasible instances were calculated. In Fig. 2, we can observe a decreasing pattern in the expected weighted distance when we increase the available budget as anticipated. This is because a smaller budget allows less relocation opportunities and when we have limited relocation opportunities the travel distance from each customer to their closest facility may increase.

We can also observe the decrease in the expected weighted distance when we increase the γ value. The effect of γ on the objective function value for each budget level can be observed better in Fig. 3. All three figures show that the objective function value decreases when we increase the value of γ as expected. Even though higher γ values may lead to less robustness for some scenarios, they allow the model to consider more location alternatives and this helps to decrease the total travel distance. This decrease becomes more apparent for higher budget levels because a higher budget gives more flexibility for relocations, which allows one to find solutions with lower travel distances.

These figures help us to determine the trade-off between the objective function value and the γ value as well as the different budget levels. We can observe that, the more available budget we have or the less robustness we seek, the smaller our expected weighted distance will be. On the other hand, budget has an impact on the trade-off between the objective function value and the γ value. When there is a small amount of available budget, the γ value does not have too much effect on the objective function value because of limited relocation opportunities, i.e. an optimal solution for $\gamma = 0.15$ may be the only feasible solution for $\gamma = 0.2$ or 0.25. When there is an ample amount of budget, the objective function value decreases as the γ value increases because there are many relocation alternatives.

5. Summary and future work

In this paper we introduced the facility relocation problem under uncertainty that considers uncertain demand changes. The objective is to minimize the expected weighted distance while making sure that relative regret for each scenario is no more than γ. As we discussed in Section 1, there are only few approaches that consider relocation of facilities, which is necessary to handle demand changes. Therefore, we presented a method that determines optimal relocations of facilities with respect to γ-robustness under uncertain demand changes.

We developed an integer programming formulation of the problem and analyzed its properties. Proving that the problem is *NP-hard*, and observing the long computational time especially for larger instances, we developed a Lagrangian Decomposition Algorithm (LDA) to expedite the solution process. We then presented numerical results that compare the solution time and quality of LDA with the exact solution method. Our experiments showed that, LDA provides a significant time gain, while satisfying the desired dual gap value. LDA is the clear winner if the problem size increased because for larger scale problems, the exact method could not generate any integer feasible solution for hours of run.

We conducted an analysis that shows the impact of budget and γ values on the expected weighted traveling distance. The objective function values decrease when we have more available budget for facility relocations. When we decrease the γ value meaning that the less robustness we desire, the expected weighted distance decreases. Therefore, this analysis help us to determine the trade-off between the expected traveling distance from customers to their closest facilities and robustness for various budget levels.

In the γ-RFRP problem, we did not consider any capacity limitations for the facilities. As a future work, this problem can be extended to a capacitated γ-RFRP to better reflect the reality. Capacity limitations will contribute to the infeasibility of the

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problems caused by robustness and budget constraint. Therefore, infeasibility issues should further be investigated and solution algorithms should be developed.

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