Optimal Egress Time Calculation and Path Generation for Large Evacuation Networks

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Abstract

Finding the optimal clearance time and deciding the path and schedule of evacuation for large networks have traditionally been computationally intensive. In this paper, we propose a new method for finding the solution for this dynamic network flow problem with considerably lower computation time. Using a three phase solution method, we provide solutions for required clearance time for complete evacuation, minimum number of evacuation paths required for evacuation in least possible time and the starting schedules on those paths. First, a lower bound on the clearance time is calculated using minimum cost dynamic network flow model on a modified network graph representing the transportation network. Next, a solution pool of feasible paths between all O-D pairs is generated. Using the input from the first two models, a flow assignment model is developed to select the best paths from the pool and assign flow and decide schedule for evacuation with lowest clearance time possible. All the proposed models are mixed integer linear programing models and formulation is done for *System Optimum (SO)* scenario where the emphasis is on complete network evacuation in minimum possible clearance time without any preset priority. We demonstrate that the model can handle large size networks with low computation time. A numerical example illustrates the applicability and effectiveness of the proposed approach for evacuation.

Keywords: Short notice evacuation, evacuation planning, dynamic network flow problem, minimum cost flows.

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1 Introduction

Evacuations are more common than many people realize. According to FEMA [1], hundreds of times each year, transportation and industrial accidents release harmful substances, forcing thousands of people to leave their homes. Fires, floods and other natural disasters frequently cause evacuations. Almost every year, people along the Gulf and Atlantic coasts of the United States evacuate in the face of approaching hurricanes. When community evacuation becomes necessary in light of an approaching danger, emergency managers face a set of logistical and action timing decisions. Decisions concerning pre-positioning of personnel and material, mobilization of resources, decision on evacuation routes and schedules, distribution of humanitarian aid, communication of advisory messages, and updating of supplies, all of these are inter-dependent and become the task of prime importance.

Emergency evacuation is the immediate and rapid movement of people away from the threat or actual occurrence of a hazard. Since population is mostly concentrated in cities, it becomes imperative for the disaster management personnels to have an efficient plan that could safely evacuate its residents to a shelter location and avoid the chaos. Heightened interest in emergency management and evacuation operations has impelled transportation professionals to focus on evacuation modeling and planning. Mass exodus calls for effective and efficient route planning and schedule allocation considering the spatial and temporal constraints in situations of road congestion, blockage or otherwise inaccessibility due to other dangerous circumstances.

A situation where the priority is to completely evacuate an area demands a comprehensive and non-complex system optimal *(SO)* solution for the path, schedule and corresponding flow that can be easily followed and implemented by the authorities. In no-notice evacuation scenarios, the preparation or readiness for the execution of the plan is curtailed which is critical for a successful evacuation effort. Responders will be unable to pre-activate or pre-position resources in preparation for the specific situation mandating the evacuation. Moreover, there are limited number of personnel with particular skills and knowledge and also limitation on the tools to determine and monitor the status of the transportation network whose absence or shortage can significantly hinder the evacuation operation. Such situation demands for a plan that use minimum resource and still is equivalently effective in carrying out the evacuation.

Managerial focus during such an emergency scenario is to get answers for two imposing questions, the minimum number of paths required for routing the evacuation traffic and the total clearance time required for complete evacuation. Finding answers to these questions is very important so that all the necessary (often limited) resources required for evacuation can be channelized quickly, thus, helping the emergency managers to smoothly execute the planned exit strategy. In this paper, we provide emergency managers with a comprehensive solution that will help them in decision making during evacuation scenarios. In particular, we deliver to emergency managers a complete evacuation plan for completely evacuating a large area, a lower bound to the clearance time required for complete evacuation and the minimum number of paths to be loaded with evacuating traffic that will be sufficient to achieve the limiting value of the clearance time or be close to the lower bound of the clearance time.

1.1 Literature Review

Efficient regional evacuation process is dependent on various factors such as an accurate description of transportation infrastructure, population distribution, vehicle utilization and behavior of evacuees like response time and route selections [2]. Previous research on the subject of evacuation route selection devised strategies to ensure smooth and quick evacuation process. Various algorithms based on discrete time dynamic network flow models have been proposed. Ford and Fulkerson [3] first came up with maximum dynamic network flow problem for a single source and sink node, and presented a flow augmentation method to solve this problem in polynomial time. Although, their method can be expanded to multiple source and sink nodes [4], it is not applicable for evacuation networks where there is a finite amount of supplies in source node and limited capacity in sink nodes. For efficient use of infrastructure, Cova and Johnson [5] used network flow model for the lane based evacuation routing. Kisko et al. [6] used minimum cost dynamic network optimization to find optimal clearance time and possible bottleneck locations for building evacuation. There are other lines of research for evacuation that concentrate on problems such as shelter locations and shelter assignments, capacity expansions of road networks and lane reversibility popularly known as contra-flow that address the infrastructure limitation. But the problem of finding minimum evacuation paths to address limited resource has never been addressed in evacuation planning literature.

Linear programming (LP) methods are often used to solve the dynamic network flow models and an extensive literature review of these models and algorithms specifically for evacuation problems can be found in [7, 8, 9]. Algorithms in evacuation literature using linear programming approach start with representing the transportation infrastructure using a network graph $\mathscr{G}$, then assigning an estimated upper bound T for the clearance time, constructing a time-expanded network $\mathscr{G}_T$ [7] and finally, solving for optimal as minimum cost dynamic network flow problem. The linear programming approach for the evacuation models finds an optimal solution for small and moderate sized networks, but fails to scale up for large urban transportation networks. Hoppe and Tardos [10, 4, 11] gave some polynomial time algorithm for optimal solution of minimum cost network flow problem, but the performance is still poor and has little practical importance [12]. Current evacuation modeling research is, therefore, directed towards heuristic solutions.

Large network capacity constrained routing algorithms based on maximum dynamic network flow models have also been proposed and solved using heuristic methods [13, 14, 15, 16]. Although these papers are using heuristic methods, they still take considerably long time to find solution for large size capacity constrained transportation networks. Analysis of the models used for evacuation traffic scheduling based on dynamic network flow points to following limitations. First, the computational run-time is very high for the tremendously increased network size of a large transportation network that finds solution based on time-expanded network $\mathscr{G}_T$. Algorithms based on time-expanded network are considered as pseudo-polynomial algorithm [4] depending polynomially on T. Second, upper bound T for the clearance time is estimated by the user and given as an input to the model. This estimation of T is inaccurate and will result in under or over estimation both having their own adverse consequences. Finally, minimizing the number of evacuation paths and still having the same throughput from the network has never been addressed in literature. This paper marks the first attempt in this direction.

A bi-objective dynamic network flow model is formulated to find the least number of evacuation paths for complete evacuation within the minimum clearance time. The optimization model is a mixed integer nonlinear programming model (MINLP) with multiple objectives and such problems are often difficult to solve. Therefore, a three phase solution method is proposed for this problem by decomposing the original model into three separate sub-models. The solutions of these models provide a lower bound on clearance time for complete evacuation, a solution pool of feasible paths and the minimum number of paths required for evacuation in least possible time along with the starting schedules on the selected paths assuming a variable flow rate on the paths at each time interval. Path schedule and the corresponding flows on those paths are assigned according to an established and controllable evacuation plan based on a dynamic transportation network. For a situation when the emergency management has only limited number of resources to allocate and, therefore, they want to limit the number of paths to be used for evacuation to a fixed value, we provide solutions for the clearance time and the corresponding evacuation traffic flow and schedule that is channelized through those limited number of paths.The proposed models are mixed integer linear programing models and formulation is done for System Optimum (SO) scenario where the emphasis is on complete network evacuation in minimum possible clearance time without any preset priority. We demonstrate that the model can handle large size networks with low computation time.

The rest of the paper is organized as follows. In Section 2, an overview is provided for the construction of transportation network on a graph and then the model is formulated for the multi-objective evacuation routing and scheduling problem. Section 3 introduces the solution framework along with the corresponding models in the subsequent subsections. These subsections discuss the Integer Programming (IP) formulations for the SO minimum cost dynamic network flow problem for clearance time calculation, shortest path formulation for generation of solution pool of paths between all O-D pairs and a combinatorial problem for selecting minimum number of paths to be used for complete evacuation and assigning flow values on those paths. The applicability of the model is demonstrated with a sample network in Section 4 along with the discussion of results for various scenarios that might occur during evacuation. Conclusions and future works are presented in Section 5.

2 Minimum Time Least Path Model

A dynamic network flow model has been used to mathematically represent traffic flow evolution in an evacuation network for the proposed optimization model. A dynamic network consists of a graph with capacities and transit times associated with each arc. A dynamic network can be visualized as a static network with an additional dimension representing time, i.e., static network is repeated for each discrete slice of time. Traffic assignment on such time-expanded networks relies upon a more aggregate representation of traffic as a series of flows that attempts to match the demand for road space with the capacity of the highway system's links and intersections at various time.

The model is having two objectives and can be stated as a summation of scaled value of clearance time and path counts for the number of paths to be used for evacuation. We name the model as minimum time least path (MTLP) model. Mathematical notation used to describe the model are mentioned below.

Table 1: Notation

Notation	Description
$\mathscr{N}$	Set of all nodes
$\mathscr{N}_a$	Set of all source nodes
$\mathscr{N}_s$	Set of all sink nodes
$\mathscr{N}_i$	Set of all intermediate nodes
$\mathbb{T}$	Set of time periods $\mathbb{T} = \{0, 1, \ldots, T-1\}$
$\mathscr{A}$	Set of all arcs
$\mathscr{A}(i)$	Set of arcs going out of node i
$\mathscr{A}^{-1}(i)$	Set of arcs coming into node i
ξ_i	Initial number of vehicles in a node i
$\mathscr{C}_j$	Capacity of sink node j
$\mathscr{U}_{ij}$	Maximum capacity of arc from node i to j
$\mathbb{O}_p$	Source node of path p
$\mathbb{D}_p$	Sink node of path p
θ_{ijp}	Transit time from origin of path p to arc (i,j)
κ	Scaling factor
M_p	Number of vehicles at the origin node of path p

There are three decision variables in MTLP model:

$f_{pt} \in \mathbb{Z}^+$: number of vehicles flowing on path $p \in \mathscr{P}$ at any discrete time $t \in \mathbb{T}$,

$$y_{i,j,p} = \begin{cases} 1, & \text{if path } p \text{ uses arc } (i,j); \\ 0, & \text{otherwise.} \end{cases} \qquad \forall p \in \mathscr{P}, \forall i \in \mathscr{N}, \forall j \in \mathscr{A}(i).$$

$$w_p = \begin{cases} 1, & \text{if the path } p \text{ is selected;} \\ 0, & \text{otherwise.} \end{cases} \qquad \forall p \in \mathscr{P}.$$

Formulation of the optimization problem is based on an arc-based model [14]. Objective function (1) is a linear combination of two separate objectives, i.e., $z = \kappa \cdot z_a + z_b$. Optimal value for the objective $z_a^* = min\{\sum_{p \in \mathscr{P}} w_p | \text{Constraints}\}$ corresponds to minimum number of selected paths w_p

$$
\begin{aligned}
\text{Minimize} \quad & \kappa \sum_{p \in \mathscr{P}} w_p + \sum_t \sum_p t \cdot f_{pt} && && (1)\\
\text{Subject to:} \quad & \sum_{j|(i,j)\in \mathscr{A}(i)} y_{ijp} - \sum_{j|(j,i)\in \mathscr{A}^{-1}(i)} y_{jip} = 1, && \forall p \in \mathscr{P}, i \in \mathscr{N}, i = \mathbb{O}_p, && (2)\\
& \sum_{j|(i,j)\in \mathscr{A}(i)} y_{ijp} - \sum_{j|(j,i)\in \mathscr{A}^{-1}(i)} y_{jip} = 0, && \forall p \in \mathscr{P}, i \in \mathscr{N}, i \neq \mathbb{O}_p, i \neq \mathbb{D}_p, && (3)\\
& \sum_{j|(i,j)\in \mathscr{A}(i)} y_{ijp} - \sum_{j|(j,i)\in \mathscr{A}^{-1}(i)} y_{jip} = -1, && \forall p \in \mathscr{P}, \forall i \in \mathscr{N}, i = \mathbb{D}_p, && (4)\\
& \sum_{p \in \mathscr{P}} f_{p(t-\theta_{ijp})} \cdot y_{ijp} \leq \mathscr{U}_{ij}, && \forall (i,j) \in \mathscr{A}, \quad t \in \mathbb{T}, && (5)\\
& \sum_{p|\mathbb{O}_p = i} \sum_{t \in \mathbb{T}} f_{pt} = \xi_i, && \forall i \in \mathscr{N}_a, && (6)\\
& \sum_{p|\mathbb{D}_p = j} \sum_{t \in \mathbb{T}} f_{pt} \leq \mathscr{C}_j, && \forall j \in \mathscr{N}_s, && (7)\\
& \sum_{t \in \mathbb{T}} f_{pt} \leq M_p \cdot w_p, && \forall p \in \mathscr{P}, && (8)\\
& f_{pt} \in \mathbb{Z}^+, w_p, y_{ijp} \in \{0,1\} && \forall p \in \mathscr{P}, \quad \forall t \in \mathbb{T}, \quad \forall (i,j) \in \mathscr{A}. && (9)
\end{aligned}
$$

required for complete evacuation. Optimal $z_b^* = min\{\sum_t \sum_p t \cdot f_{pt} | \text{Constraints}\}$ corresponds to minimum total time required for complete evacuation. Since the two objective functions are in different scales, a constant factor κ is multiplied to z_a to bring it to a scale similar to z_b.

Path generation is done using the standard set of network flow balance equations as specified by constraints (2)-(4). These set of equations are repeated for each path originating from a particular source node and ending into corresponding sink node and thus deciding on the arcs to be included into the set of optimal paths. Constraint (5) is the capacity constraint on arc (i, j) at each time t. This constraint limits the total flow using all paths $p \in \mathscr{P}$ at any particular time t on any arc $(i, j) \in \mathscr{A}$ to the maximum capacity of that arc. Variable $f_{p(t-\theta_{ijp})}$ ensures that the flow originating at path p at time $t - \theta_{ijp}$ reaches arc (i, j) after the transit time θ_{ijp}, thus, allowing multiple paths to simultaneously share the same arc. Using constraint (6), we balance the total outgoing flow from source node $i \in \mathscr{N}_a$ over the time horizon $t \in \mathbb{T}$ to the original supply of the source node. Similarly, constraint (7) limits the flow reaching at the sink nodes below or equal to the maximum holding capacity of the destination node $j \in \mathscr{N}_s$. Constraint (8) is introduced to relate the flow on any path p over all time $t \in \mathbb{T}$ with the path selection variable w_p. Parameter M_p limits the flow to the original number of supply at the origin of path if the path is selected in the solution. Otherwise, the flow is set to zero for that path.

We observe that constraint (5) is quadratic. Reformulation-Linearization Technique (RLT) by Sherali and Tuncbilek [17] is applied to reformulate the problem to a linear model and take advantage of linear optimization solvers. RLT works by introducing a new variable $x_{ijp(t-\theta_{ijp})} = f_{p(t-\theta_{ijp})} \cdot y_{ijp}$. Modification of the quadratic constraint (5) is done by replacing the 2^{nd} order non-linearity with the new variable $x_{ijp(t-\theta_{ijp})}$ and also adding the bounds corresponding to the new variable in our earlier formulation to come up with the new set of constraints (10) - (12).

$$\sum_{p \in \mathscr{P}} x_{ijp(t-\theta_{ijp})} \leq U_{ij}, \qquad \forall (i,j) \in \mathscr{A}, \quad t \in \mathbb{T}, \tag{10}$$

$$0 \leq x_{ijp(t-\theta_{ijp})} \leq f_{p(t-\theta_{ijp})}, \tag{11}$$

$$f_{p(t-\theta_{ijp})} - M(1 - y_{ijp}) \leq x_{ijp(t-\theta_{ijp})} \leq M \cdot y_{ijp}. \tag{12}$$

Using this transformation, $x_{ijp(t-\theta_{ijp})}$ can take a value of either zero or it can take a value of positive integer value $f_{p(t-\theta_{ijp})}$. This arc-based linear model can be solved to achieve the multi-objective result. The model gives an optimal solution for a very small sized network but it is not able to scale up for even a medium sized network. Linearization of the model required the introduction of new variables as well as a new set of constraints which makes the model very large and consequently it is very difficult to solve. Moreover, there is an inherent difficulty of choosing an appropriate weight κ applied to the objective of minimizing the number of paths. Therefore, we present a different approach that can achieve the desired multiple objectives in the next section.

3 Three Phase MTLP

The proposed approach decompose the problem into three separate problems and takes three steps in achieving the bi-objectives and is referred as three phase MTLP hereafter in the paper. To make the model description more modular, this section is divided into three different subsections each discussing their corresponding mathematical model and solution method. A minimum cost flow model is first used to come up with a lower bound for clearance time that is required to completely evacuate the network. This model is named as Minimum Egress Time (MET) model and is discussed in Section 3.1. This lower bound on clearance time is used as an input to the model that is used for coming up with flow and schedule. A shortest path model on a static network is used to find a pool of evacuation paths from each source node towards a sink node and is discussed in Section 3.2. We name this as the Path Set Generation (PG) model. Any particular path in the set of paths from each source node can be a possible candidate for final route selection for evacuation. During an evacuation scenario, this set of paths can be provided by the emergency managers keeping account of all the usable paths for evacuation and discarding the paths that might be flooded or not suitable for handling traffic. After finding the set of paths and the lower bound on clearance time, a flow generation model is used as a combinatorial problem to select the minimum number of paths from the path pool, assign the flow on those paths and come up with the schedule for the usage of those paths required for complete evacuation. Section 3.3 has a detailed discussion on this model which we term as Path Selection and Flow Generation (PSFG) model.

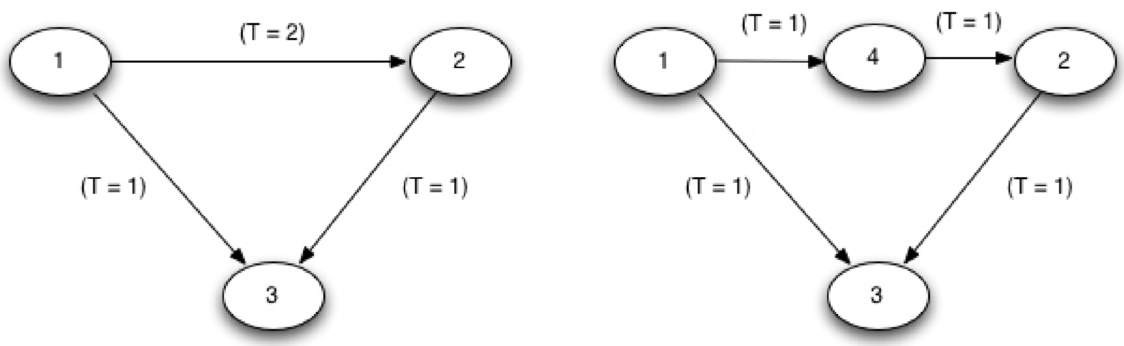
3.1 Generation of Lower Bound on Egress Time

In our model, we assume that the complete information of the initial loading of the transportation network is available along with a network graph representing the transportation infrastructure, i.e., the nodes representing the intersections and arcs representing the links on the road along with travel time and link capacity. A minimum cost flow model is designed that can push entire evacuees out of the network in minimum possible time. Before starting the description of the MET model, we introduce notation and network modifications for our formulation.

3.1.1 Notation and Network Modifications

Consider a static network $\mathscr{D} = (\mathscr{N}, \mathscr{A})$ that represents the transportation network for the area of evacuation. The set of nodes $\mathscr{N}$ are classified into impact nodes $\mathscr{N}_a$, intermediate nodes $\mathscr{N}_i$ and sink nodes $\mathscr{N}_s$. Impact nodes are the nodes where the vehicles are waiting and are ready for evacuation. Sink nodes $\mathscr{N}_s$ are the safe destination nodes where they are finally headed to. Intermediate nodes $\mathscr{N}_i$ represents the intersections in the road network and vehicles are not allowed to stay at these nodes. Primary goal of the model is to find the minimum clearance time for the underlying network with a given initial supply of vehicles. Therefore, we wish the network to have nodes separated by unit transit time such that it is possible to capture the precise time till there is flow in the network.

Network modification is achieved by introducing dummy nodes between two nodes if the transit time on the arc connecting them is more than unity. Dummy arcs are introduced to connect to these nodes which now have unit transit time and have the arc capacity same as it was for the parent arc. The above modification is better illustrated by a sample network $\mathscr{D}_o = (\mathscr{N}_o, \mathscr{A}_o)$ with three nodes and three arcs as shown in Figure 1a. The boxed units denote the transit time on arcs and is greater than one for the arc from node 1 to node 2. The sample network $\mathscr{D}_o$ is modified by introducing a dummy node with unit arc transit time between the adjacent nodes as shown in Figure 1b. This results in a new graph $\mathscr{D}_m = (\mathscr{N}_m, \mathscr{A}_m)$ with nodes separated by unit transit time. Network $\mathscr{D}_m$ is the modified network that is being used in the proposed model.



(a) Original Network: $\mathscr{D}_o = (\mathscr{N}_o, \mathscr{A}_o)$

(b) Modified Network: $\mathscr{D}_m = (\mathscr{N}_m, \mathscr{A}_m)$

Figure 1: Network modification

We define the following constants for each node: $\mathscr{C}_i$ as the maximum number of vehicles that can stay at destination node i, and ξ_i as the initial number of vehicles at source node i. Number of vehicles present at node i at time instance t is denoted by x_i^t. The set of arcs $\mathscr{A}$ represents the connection between the node pairs. They are denoted by their starting and ending node in the network. An arc set is classified into the following three categories: a set of all outgoing arcs from a node $\mathscr{A}(i)$, a set of all arcs $\mathscr{A}^{-1}(i)$ coming into a node, and $\mathscr{A}(\mathscr{N}_s)$ for the arcs that are coming into sink nodes. Associated with arcs are transit time t_{ij}, maximum flow capacity $\mathscr{U}_{ij}$ and actual flow y_{ij}^t moving from node i to node j at any time instant t. We denote as $\mathbb{T}$, the set of discrete time intervals, i.e., $\mathbb{T} = \{0, 1, \ldots, T-1\}$.

The initial number of vehicles ξ_i of all the nodes $i \in \mathscr{N}$ constitute the boundary condition assuming the network is not initially loaded. The values of parameter ξ_i are set according to the initial traffic condition at the beginning of the time period of interest. Vehicles are not allowed to stay at the intermediate nodes ($x_i^t = 0, \forall i \in \mathscr{N}_i, \forall t \in \mathbb{T}$) thus making the problem as a capacitated network flow problem. For the problem formulation, we further introduce another dummy node '$\mathscr{N}_d^+$' to the modified network graph $\mathscr{D}_m$ and name that as "Super-sink". Super-sink is connected with all destination nodes and is set to have an infinite capacity ($\mathscr{C}_{\mathscr{N}_d^+} = \infty$). Arcs joining the sink nodes to the super-sink node allow infinite flow in zero transit time thus not limiting the super-sink node with any constraints.

A *SO* MET model with multiple source and single destination is mathematically formulated based on the modified network graph. Solution to this model gives us the lower bound for clearance time. A detailed discussion on MET model is presented in the next section.

3.1.2 Optimization Model

To achieve the *SO* objective of pushing the maximum flow towards the destination in minimum possible time, we propose a minimum cost dynamic network flow formulation for the evacuation planning problem. Two integer decision variables y_{ij}^t and $x_i^t \in \mathbb{Z}^+$ are introduced for the MET model. Integer variable x_i^t denotes the number of vehicles present at node i at any time t and integer variable y_{ij}^t denotes the number of vehicles leaving node i towards node j at time t. The objective of *SO* dynamic network flow evacuation problem is to minimize the total travel time experienced collectively by all the users in the system. Therefore, the choice of destination node is not at the discretion of the individual vehicle but is decided by the model that results in optimum evacuation for the overall system.

For a dynamic network flow model, the travel time experienced by users in the network is equivalent to the difference between the arrival time at the destination and departure times from the source for every unit of flow within the network. Since the departure times are known and constant, they can be dropped and the arrival times can be considered as an equivalent travel time that has to be minimized. Arrival times can be determined when the flow exits the network. In order to minimize the total travel time in the network, we assign an uniformly increasing cost t to the flow expression departing the network. This will assign an increasingly higher cost to the delayed flow terminating into the destination. The objective of reducing the total travel time in the network can, therefore, be expressed as a product of cost function and flow function summed over all time for the flow exiting the network:

$$\sum_{t\in\mathbb{T}} \sum_{(i,j)\in\mathscr{A}(N_s)} t \cdot y_{ij}^t. \tag{13}$$

MET model for finding the lower bound of the clearance time is formulated below.

$$\begin{aligned}
\text{Minimize:}\quad & \sum_{(i,j)\in\mathscr{A}(\mathscr{N}_s)} \sum_{t\in\mathbb{T}} t \cdot y_{ij}^t && && (14)\\
\text{Subject to:}\quad & x_i^0 + \sum_{(i,j)\in\mathscr{A}(i)} y_{ij}^0 = \xi_i && \forall i \in \mathscr{N}, && (15)\\
& x_i^t - x_i^{t-1} + \sum_{(i,j)\in\mathscr{A}(i)} y_{ij}^t - \sum_{(j,i)\in\mathscr{A}^{-1}(i)} y_{ji}^{t-1} = 0 && \forall t \in \mathbb{T}\setminus\{0\},\quad \forall i \in \mathscr{N}, && (16)\\
& x_i^t - x_i^{t-1} - \sum_{(j,i)\in\mathscr{A}^{-1}(i)} y_{ji}^t = 0 && \forall t \in \mathbb{T}\setminus\{0\},\quad i = \mathscr{N}_d^+, && (17)\\
& \sum_{(j,i)\in\mathscr{A}^{-1}(\mathscr{N}_d^+)} \sum_{t\in\mathbb{T}} y_{ji}^t = \sum_{i\in\mathscr{N}} \xi_i, && && (18)\\
& \sum_{t\in\mathbb{T}\setminus\{0\}} \sum_{(j,i)\in\mathscr{A}^{-1}(i)} y_{ji}^{t-1} \le \mathscr{C}_i && \forall i \in \mathscr{N}_s, && (19)\\
& y_{ij}^t \le \mathscr{U}_{ij} && \forall t \in \mathbb{T},\quad \forall (i,j) \in \mathscr{A}, && (20)\\
& x_i^{|T|-1} = 0 && \forall i \in \mathscr{N}, && (21)\\
& 0 \le x_i^t \le \mathscr{C}_i && \forall t \in \mathbb{T},\quad \forall i \in \mathscr{N}, && (22)\\
& y_{ij}^t \ge 0 && \forall t \in \mathbb{T},\quad \forall (i,j) \in \mathscr{A}, && (23)\\
& y_{ij}^t \in \mathbb{Z}^+, x_i^t \in \mathbb{Z}^+. && && (24)
\end{aligned}$$

As in any network model, the movements of vehicles between nodes is defined by flow propagation and flow conservation equations. These relations decide respectively the flows y_{ij}^t between two nodes based on upstream/downstream traffic flow on the arcs and depict the evolution of the node status (i.e., the number of vehicles in each node x_i^t) over time. Considering a time-expanded network, y_{ij}^t can be visualized as the flow on transition arcs and x_i^t as the flow on holdover arcs associated only with the source nodes. Constraint (15) is the flow balance for the network constraint at the start time of the analysis. Constraint (16) takes over the flow balance constraint after the network loading at $t = 0$ is completed. Constraint (17) is the equivalent of constraint (16) for the super-sink node over all time $t \in \mathbb{T}$. These relations find the flow on an arc between two nodes that is bounded by the arc capacity and the supply and capacity of nodes. Constraint (18) states that the total incoming flow into the super-sink node should be equal to the total supply at the start of the analysis period. The implication of this constraint is that it does not allow any withholding at the impact nodes and thereby pushing for the complete evacuation of the network. It pushes the flow towards the sink nodes which are connected to the super-sink node and thus result in flow propagation in the network. The total amount of flow, however, is determined by the objective function (14). Constraint (19) specifies that the total incoming flow into the set of sink nodes $i \in \mathscr{N}_s$ should not exceed the capacity of the sink nodes. Limiting constraint on the maximum flow possible on any arc in the arc set $(i, j) \in \mathscr{A}$ at any time $t \in \mathbb{T}$ is expressed in constraint (20). Constraint (21) specifies the model to push for zero vehicle that are left behind at the end of the analysis period. Constraints (22) and (23) state the non-negativity conditions with an additional condition of limiting the node capacity x_i^t to its maximum capacity $\mathscr{C}_i$. It should be noted that the initial assignment period T should be high such that all traffic assigned in the network exits the network, otherwise they would be left behind and problem will not meet the constraints.

Theorem 1 *The optimal solution of MET gives a lower bound for the minimum clearance time of the MTLP model, i.e.,* $z^*_{MET} \leq z^*_b$.
Proof: MET model is a relaxation of MTLP model. If we remove from the MTLP model, the path generation constraints as well as constraint (8) *for path selection, it reduces to the MET model. Therefore,* $z^*_{MET} \leq z^*_b$.

Using the above formulation, we exploit the property of unit travel time between nodes in the network and formulate the problem as a *SO* minimum cost network flow problem. Separation of the nodes by unit time allows the model evolution for flow that can be determined at each time unit. Solution of the proposed MET model is used to calculate the lower bound on clearance time T required for complete evacuation.

3.2 Paths Set Generation

The main objective of the paths set generation model (PG) is to find a set of feasible paths from a source node to a sink node. In an evacuation scenario, emergency personnels generally prefer to use the paths that are prescribed for evacuation. Situations might arise when the prescribed path is not usable or would not be able to handle the traffic surge during emergency to evacuate within safe time. Creating a pool of usable paths for evacuation would provide a viable alternative to emergency managers where they can set a priority for the paths to be used. Generation of this set of possible paths is achieved using the solution pool feature of CPLEX [18]. This feature allows to generate and store multiple solutions in addition to the optimal solution for path set generation model.

Paths set generation model is expressed using same notations as used in earlier models. PG model is a shortest path problem that we use for finding the paths between all O-D pairs for a static network $\mathscr{D} = (\mathscr{N}, \mathscr{A})$. A binary decision variable y_{ij} in the path generation model takes a value of 1 if the arc is present in the shortest path between node i and node j and takes a value of 0 if the arc is not present. Notation t_{ij} denotes the transit time from node i to node j. Using this model, we aim to find the shortest path from the source node $i \in \mathscr{N}_a$ to the super sink node '$\mathscr{N}_d^+$'. Therefore, model objective (25) is minimizing the total transit time on arcs if that arc is present in the path from origin to destination. Transit time on arc from i to j is expressed as t_{ij}, and it is given as an input to the model. Constraint (26) ensures that the vehicle leaves the origin by selecting an arc from the source node. Constraint (27) is for intermediate nodes which ensures that if the vehicle enters the node then it must leave the node as well. Constraint (28) ensures that the vehicle reaches the destination $\mathscr{N}_d^+$. Constraints (29) and (30) limit the total number of arcs going out and coming into a node to 1 and thus eliminating the generation of cycles in the solution pool. This problem has as many variables as the number of arcs in the network.

Using the solution pool feature, an appropriate number of paths can be set to be populated for the solution. A user defined set of paths are selected from this solution that is generated using a solution pool relative gap of α. This relative gap allows the paths to be generated that are within $100\alpha\%$ of the incumbent solution, i.e., paths are generated with travel times ranging from least time required between the O-D pair to $100\alpha\%$ of the least travel time. Solution pool feature might generate duplicate paths which are discarded and a repository of unique paths is considered to be used for evacuation.

3.3 Path Selection and Flow Generation

Having known the lower bound for clearance time and the set of paths that can be used for evacuation, we use the Path Selection and Flow Generation (PSFG) model to choose the best

$$
\begin{aligned}
\text{Minimize:}\quad & \sum_{(i,j)\in\mathscr{A}} t_{ij}\cdot y_{ij}, && && (25)\\
\text{Subject to:}\quad & \sum_{j|(i,j)\in\mathscr{A}(i)} y_{ij} - \sum_{j|(j,i)\in\mathscr{A}^{-1}(i)} y_{ji} = 1, && i\in\mathscr{N}_a; && (26)\\
& \sum_{j|(i,j)\in\mathscr{A}(i)} y_{ij} - \sum_{j|(j,i)\in\mathscr{A}^{-1}(i)} y_{ji} = 0, && \forall i\in\mathscr{N}\setminus\{\mathscr{N}_a\cup\mathscr{N}_d^+\}; && (27)\\
& \sum_{j|(i,j)\in\mathscr{A}(i)} y_{ij} - \sum_{j|(j,i)\in\mathscr{A}^{-1}(i)} y_{ji} = -1, && i=\mathscr{N}_d^+; && (28)\\
& \sum_{j|(i,j)\in\mathscr{A}(i)} y_{ij} \le 1 && \forall i\in\mathscr{N}, && (29)\\
& \sum_{j|(j,i)\in\mathscr{A}^{-1}(i)} y_{ji} \le 1 && \forall i\in\mathscr{N}, && (30)\\
& y_{ij}\in\{0,1\}. && && (31)
\end{aligned}
$$

possible set of paths from each source node and find the flow and schedule on those paths. This model is a combinatorial problem where we choose the best paths from the pool of available solutions obtained from path generation model. It should be noted that the bound on the clearance time was calculated using MET based on minimum cost flow model without any prior path information. Since the objective of MET is to push the maximum flow in minimum time, model tries to squeeze all the flow in minimum possible time by pushing the flow on arcs to the maximum arc capacity at all times. Consequently, it might not be possible to assign flows on limited path set within given time T. Algorithm 1 describes the process that we use to come up with our results.

Algorithm 1 Flow Generation Algorithm

```
repeat
  SolutionStatus ← Solve PSFG : f_pt, y_p, T
  if (SolutionStatus == Infeasible) then
    T++;
  end if
until (SolutionStatus == Feasible)
```

In the flow generation algorithm, clearance time T is initialized with the lower bound of the clearance time obtained from the MET model and the path pool $\mathscr{P}$ is initialized from solution of the PG model. These initial values are given as an input to the PSFG model. If the solution to PSFG is feasible, i.e., paths are found in the pool that can empty the network within T, then flow and schedule on those paths are obtained from PSFG. If the solution is infeasible, the clearance time T is increased by one unit. The process is repeated until the feasible solution is obtained. Using the conservative approach of increasing the clearance time by unity, we ensure that the total evacuation is completed within minimum clearance time for the paths available in the solution pool.

3.3.1 Notation

Flow assignment model is a combinatorial problem that considers the flow in the network graph $\mathscr{D} = (\mathscr{N}, \mathscr{A})$ over each discrete time interval t. Mathematical notation used for defining the model for flow assignment are same as in earlier model along with few additional notations. Binary expression δ_{pa} is set to 1 if the path p contains arc $a \in \mathscr{A}$ and 0 otherwise. Parameter θ_{pa} represents the transit time from origin of path p to arc a. Parameter $\mathscr{U}_a$ denotes the maximum arc capacity. $\mathbb{O}_p$ denotes the source node of path p and $\mathbb{D}_p$ denotes the sink node of path p.

3.3.2 Flow Assignment Model

Following are the decision variables defined for the optimization model:

$f_{pt} \in \mathbb{Z}^+$ representing the flow on path $p \in \mathscr{P}$ at any discrete time $t \in \mathbb{T}$.

$$y_p = \begin{cases} 1, & \text{if the path } p \text{ is selected;} \\ 0, & \text{otherwise.} \end{cases} \qquad \forall p \in \mathscr{P}.$$

Formulation for the flow assignment model can be described using objective function (32) and constraints (33) – (37).

$$\text{Minimize} \quad \sum_{p \in \mathscr{P}} y_p \tag{32}$$

$$\text{Subject to:} \quad \sum_{p \in \mathscr{P}} f_{p(t-\theta_{pa})} \cdot \delta_{pa} \leq \mathscr{U}_a \qquad \forall a \in \mathscr{A}, \quad t \in \mathbb{T}, \tag{33}$$

$$\sum_{p | \mathbb{O}_p = i} \sum_{t \in \mathbb{T}} f_{pt} = \xi_i \qquad \forall i \in \mathscr{N}_a, \tag{34}$$

$$\sum_{p | \mathbb{D}_p = j} \sum_{t \in \mathbb{T}} f_{pt} \leq \mathscr{C}_j \qquad \forall j \in \mathscr{N}_s, \tag{35}$$

$$\sum_{t} f_{pt} \leq M_p \cdot y_p \qquad \forall p \in \mathscr{P}, \tag{36}$$

$$f_{pt} \in \mathbb{Z}^+, y_p \in \{0, 1\} \qquad \forall p \in \mathscr{P}, \quad \forall t \in \mathbb{T}. \tag{37}$$

Objective function (32) minimizes the total number of paths selected for evacuation. These paths are selected from the solution pool $\mathscr{P}$ obtained from the path generation model. Selection of the paths is based on the criteria which ensures that all the supply at origin nodes is exhausted within the given clearance time T. The objective will thus give priority to the paths that have greater capacity and lower travel time.

Constraint (33) ensure that the sum of flows for all paths p on any arc $a \in \mathscr{A}$ during any interval of time t should not exceed the maximum capacity of that arc. In this constraint, parameter δ_{pa} selects the arc constituting the path. Variable $f_{p(t-\theta_{pa})}$ ensures that the flow originating at path p at time $t - \theta_{pa}$ reaches arc a after the transit time θ_{pa}. This constraint allows the simultaneous sharing of any arc by multiple paths. Constraint (34) guarantees that the sum of flows on path originating from the nodes in $\mathscr{N}_a$ over all time is equal to the supply at that node. This constraint could also have been greater than equal to constraint but we designed this to be a tighter constraint making the MIP problem easier to solve. This constraint generates the flow in the paths that are selected for the solution. Constraint (35) ensures that the summation of flow on paths coming into the destination over all time do not exceed the capacity $\mathscr{C}_j$ of the destination nodes $\mathscr{N}_s$. Constraint (36) limits the sum of all flows over all time $t \in \mathbb{T}$ on any path p if the path is selected in the solution. If the path is not selected then the flow of the path at all times is set to 0. If the path is selected, the summation of the flows is limited to vector $M_p = \xi_{i|\mathbb{O}_p}$, i.e., the maximum possible supply initially present at the origin of the path. Constraint (37) forces the variables f_{pt} and y_p to take integer and binary values respectively. Solutions of the model will result in variable flow on each paths and also a variable flow on the same path at different time intervals. This is because we have a *SO* objective of assigning the maximum flow for 100% evacuation within minimum possible time.

4 Computational Results

In this section, we first describe our test instances. We then present results obtained using bi-objective model and three phase MTLP solution approach described in this paper. We developed our model in a C++ environment and the problem is solved using CPLEX 12.3. Experiments were made on a PC with 3.07 GHz Intel Core i7 processor having 24GB RAM and running Ubuntu

10.04.3.

For illustration purpose, we first show the complete results obtained using our model for a small network. To demonstrate the effectiveness of the proposed model, we then present a comprehensive result for a large evacuation network that is based on the map of the Greater Houston area and Galveston County, Texas.

4.1 Numerical case study

For the demonstration purpose, we employ the small network used by Lim et.al. [14] to show the essentials of the proposed model and its performance (see Figure 2). This network has three impact nodes ($\mathscr{N}_a = \{1, 2, 3\}$), five intersections, and two safe nodes ($\mathscr{N}_s = \{9, 10\}$). Each arc in the network is assigned a transit time and capacity on the arc. These values are assumed to be constant throughout the evacuation process. Our goal is to evacuate all evacuees in minimum clearance time.

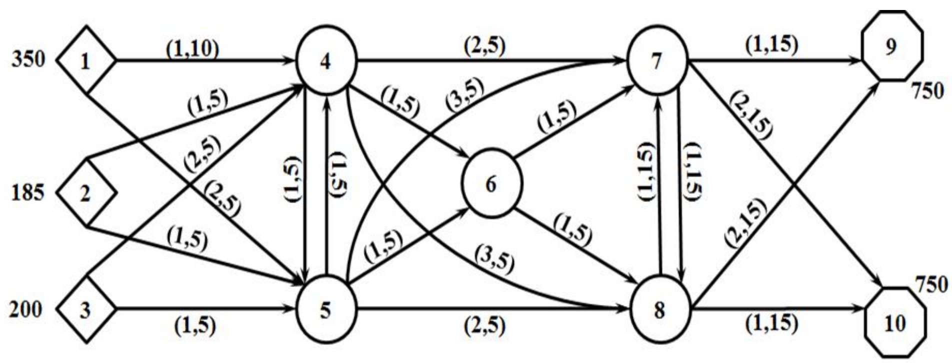


Figure 2: Evacuation Test Network

We assume that evacuation demand at time 0 (beginning of the evacuation) at impact nodes are 350, 185 and 200, respectively, and the network is not loaded initially. Destination nodes are assigned a limited capacity so as not to clog a single safe node with all the evacuees. We assign a capacity of 750 to both the destinations. We first provide a solution for the evacuation test network using three phase MTLP (Section 3). For our model compatibility, the network is modified as explained in Section 3. We provide an initial large value for T to the model as an input which should be sufficient for a complete evacuation. Note that providing a large value of T does not affect solution quality or computation time in our model at this stage. The MET model

(Section 3.1) is then solved for the modified network to find a lower bound on evacuation time to ensure 100% evacuation. The lower bound on clearance time obtained using the MET model for the test network is $T = 28$.

Generating a set of paths from each source node of the test network is done using the PG model (Section 3.2). The original test network with just the super-sink node added to the network is used for the PG model. We use a solution pool relative gap of 1 for this network. Using the solution pool method, we were able to obtain a set of 64 unique paths originating from each source node. The set of paths between all O-D pairs include the shortest path and all the paths within the relative solution gap of 100% from the optimal path. This is equivalent to including all the paths with travel time within 10 units in the solution pool if the shortest route has a travel time of 5 units between the O-D pair, i.e., paths up-to double the travel time of the minimum travel time are included in the paths pool.

Decision on selecting the best path and assigning the flow and schedules for those path is done using the PSFG algorithm (Section 3.3). The paths obtained from the PG model and the lower bound for the clearance time obtained from the MET model are fed to the PSFG algorithm as an input. PSFG algorithm is run to find the best combinations of paths. In this test network, there is no combination of paths which is able to completely evacuate within the minimum clearance time of $T = 28$. Therefore, PSFG is solved again for time $T + 1 = 29$. The selected paths are now able to take out all the supply present initially at the source nodes to the shelter or sink nodes taking minimum evacuation time of $T = 29$. The model was solved for a relative optimality gap of 5% and an absolute gap of 1. A total of 52.06 seconds was required by the CPLEX solver for finding the optimal solution for the test network.

In Table 2, we report the selected path, travel time between the O-D pair using the corresponding path and the starting schedule for those paths. Flow along the selected paths in the solution came out to be always constant for this problem with a flow value of 5. This result gives the least number of paths for the test network that is required for complete evacuation in the minimum possible time.

The bi-objective MTLP model discussed in Section 2 did not result in optimal solution for the evacuation test network in Figure 2 when the model was run for 3 hours. We tested the bi-objective model on an even smaller network with only five nodes out of which two were impact nodes, two safe nodes and a single intersection node. With a weight of $\kappa = 30$ in the bi-objective model, we

Table 2: Computational results for test network

Path	Travel Time	Evacuation Start Time
1-4-8-10	5	0 – 23
1-4-6-8-10	4	0 – 9,11 – 23
1-5-7-9	6	0 – 22
2-5-6-7-9	4	0,1,11,13 – 16, 20 – 24
2-4-7-9	4	0 – 24
3-5-8-10	4	0 – 24
3-4-5-6-7-9	6	0 – 8, 10, 15 – 17
3-4-6-8-10	5	9, 23

were able to get an equivalent solution for path and schedule of evacuation for both the models. The bi-objective model took a computation time of 0.483 seconds as opposed to 0.28 seconds by our proposed model.

Sensitivity analysis for the variation in the initial number of evacuees was done for the test network. To find out the variation in the clearance time and the corresponding number of total paths used for the modified demand, we varied the initial number of evacuees at each source node using a step size of 10% variation. Sensitivity of our model to the demand variation is shown in Figure 3. For this particular network, we observe a linear relationship of clearance time with the variation in demand at the source nodes. The number of paths used for the evacuation remains same with a value of eight for all demand scenarios for this network but this result can be different for other networks if the capacity of the arc is able to handle the reduced demand with less number of paths.

4.2 Path Limiting

Often times emergency managers are faced with the problem where they have only a limited option of paths to be selected from the set of available paths. Also, due to resource constraints, there might be a restriction on the number of paths to be used for outgoing traffic during evacuation. We introduce a new constraint of limiting the number of paths from each source node to the assigned limited value for achieving this objective. An alternative to the minimization objective of the number of paths, we can design the assignment model to have a maximum flow objective. The model with the objective of maximum flow along with a constraint for limiting the number of paths

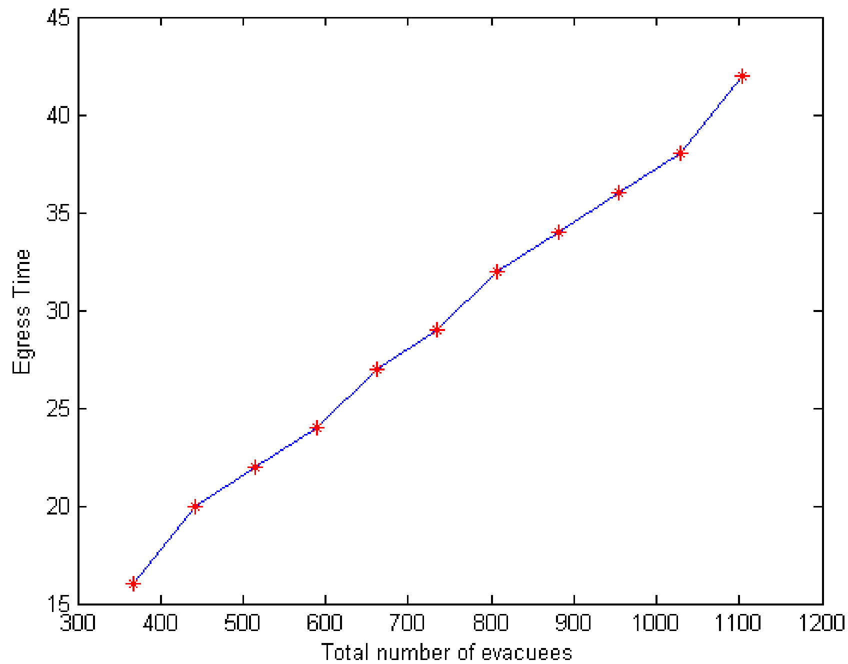


Figure 3: Sensitivity of clearance time to demand variation

from each source node can be written as follows.

$$\text{Maximize} \quad \sum_{p\in\mathscr{P}}\sum_{t\in\mathbb{T}} f_{pt} \tag{38}$$

$$\text{Subject to:} \quad \sum_{p\in\mathscr{P}} f_{p(t-\theta_{pa})}\cdot\delta_{pa} \le \mathscr{U}_a \qquad \forall a\in\mathbb{A},\quad t\in\mathbb{T}, \tag{39}$$

$$\sum_{p|O_p=i}\sum_{t\in\mathbb{T}} f_{pt} = \xi_i \qquad \forall i\in\mathscr{N}_a, \tag{40}$$

$$\sum_{p|D_p=j}\sum_{t\in\mathbb{T}} f_{pt} \le \mathscr{C}_j \qquad \forall j\in\mathscr{N}_s, \tag{41}$$

$$\sum_{t} f_{pt} \le M_p\cdot y_p \qquad \forall p\in\mathscr{P}, \tag{42}$$

$$\sum_{p|O_p=i} y_p \le \mu_i \qquad \forall i\in\mathscr{N}_a. \tag{43}$$

$$f_{pt}\in\mathbb{Z}^+, y_p\in\{0,1\} \qquad \forall p\in\mathscr{P} \quad \forall t\in\mathbb{T}. \tag{44}$$

Here again, we define the decision variables $f_{pt}\in\mathbb{Z}^+, y_p\in\{0,1\}, \forall p\in\mathscr{P}$ for the model. This model has same constraints as in the PSFG model along with constraint (43) as an additional constraint. Objective function is modified to make it a maximum flow problem. Using this model we assign the flow by considering only a limited number of paths from each source node. Constraint (43) limits the number of paths originating from the set of source nodes $\mathscr{N}_a$ to a user input value of μ_i. This model gives the flexibility to the emergency managers for choosing a limited number of paths for evacuation. Note that the number of paths may not be enough to evacuate desired evacuees within the given clearance time T. To account for the limitation for the number of selected paths to μ_i, we need to compromise on clearance time by increasing the value of T.

Figure 4 shows the result for the variation in clearance time when the number of paths are limited to a certain value for each source node. As evident, the clearance time is higher when the number of paths is very low. Clearance time decreases with the increase in number of paths and approaches a constant value. This is achieved when the limit on the number of paths originating from each source node is equal to or greater than the minimum number of paths required for evacuation within the minimum calculated clearance time.

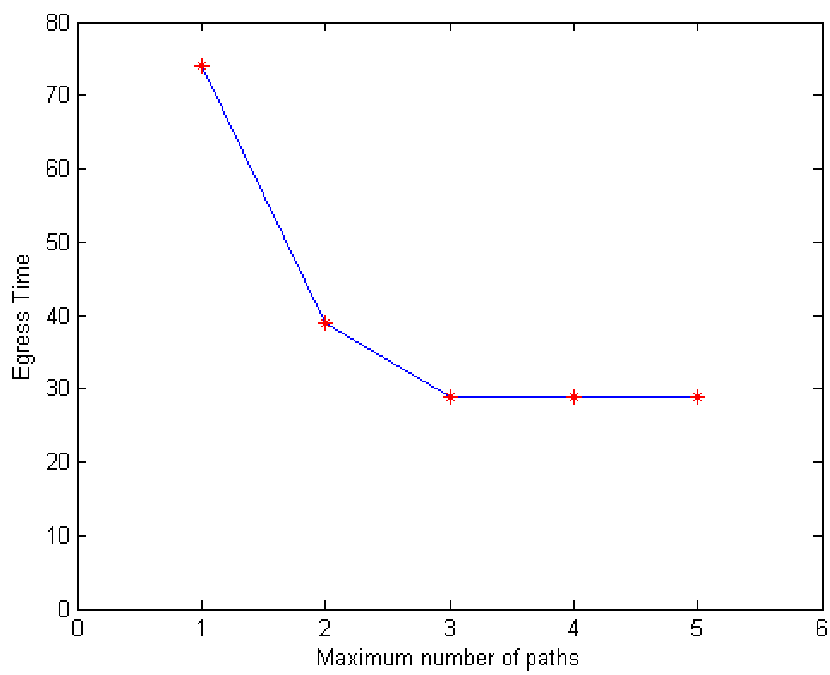


Figure 4: Sensitivity of clearance time to path limitation

4.3 Implementation on a large metropolitan area evacuation network

As discussed in literature review, one of the limitations of present network flow models is their inability to solve large-scale networks. In this section, computational experience on the network representing the Houston metropolitan transportation with a reasonable number of evacuating vehicles is presented. Houston, TX, is the fourth largest city in US and is one of the most vulnerable metropolitan cities situated at the Gulf coast. Houstonians have witnessed many hurricanes and its population being subjected to evacuation multiple times. It is the largest city that have evacuated due to hurricanes. Demonstration of the evacuation planning for the Greater Houston area would be an appropriate example for large size networks (see Figure 5).

In this evacuation network, there are 42 nodes of which nodes 1 - 13 are the source nodes and nodes 39 - 42 are the destination nodes. Arcs going out from the source nodes and coming into the destination nodes are "uni-directional" and arcs connecting the intermediate nodes are bidirectional. There are totally 566,000 vehicles distributed among the source nodes to be evacuated. Network is sampled at $\tau = 30$ minutes interval, i.e., transit time which separates each pair of node apart are multiples of τ.

Using the three phase MTLP solution approach, we were able to find a solution for 100% evacuation using a total computation time of approximately 50 minutes. Lower bound on the clearance time required for the evacuation was calculated to be 129 τ. This translates to a minimum of approximately 2 days and 17 hours required for evacuating a population of approximately 1.3 million based on the assumption that occupancy of each vehicle is 2.3 persons [2]. A solution pool of a total of 1661 unique paths between all O-D pairs was generated using a relative gap of 100% from the optimal solution. PSFG algorithm was then solved to obtain the paths, their starting schedule and corresponding flow values. The algorithm resulted to a selection of a total of 16 paths from the O-D pairs that are able to completely evacuate the network within 129 τ. Model was solved with a solution time of 1503.01 seconds for the Houston evacuation network. Complete evacuation plan for the transportation network of Houston metropolitan area can be found here [19]. Table 3 shows the paths selected between the O-D pairs along with their travel times and the total number of vehicles initially present at the source nodes.

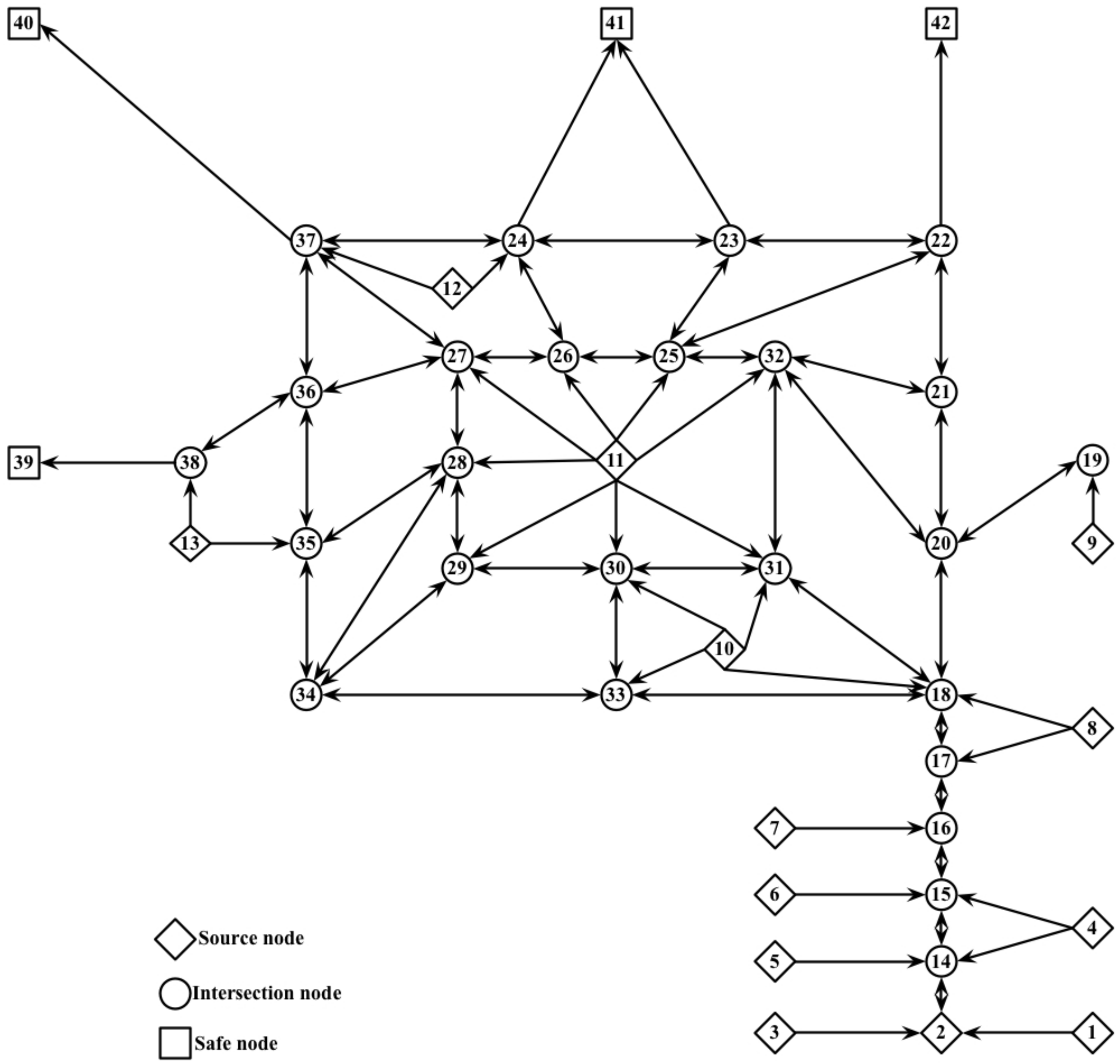


Figure 5: City of Houston Transportation Network

5 Conclusion

We developed a multi-objective network flow optimization model for evacuating a region using minimum number of evacuation paths in least clearance time possible. The concept of minimizing the number of evacuation paths is somewhat new in the evacuation literature as most papers focus on either maximizing the number of evacuees within a given clearance time or minimizing the clearance time for a given number of evacuees. Since the multi-objective formulation proved to be intractable even for a moderate sized network, the model was decomposed into multiple sub-problems and a three phase MTLP solution approach was introduced to find the desired multiple

Table 3: Computational results for Houston evacuation network

Source Node	Total Vehicles	Selected Path between O-D pair	Travel Time
1	1000	1 2 14 15 16 17 18 20 21 22 42	25
2	1000	2 14 15 16 17 18 20 32 25 22 42	25
3	1000	3 2 14 15 16 17 18 20 21 22 25 23 24 41	26
4	1000	4 14 15 16 17 18 20 21 32 25 23 24 41	25
5	1000	5 14 15 16 17 18 33 34 35 36 27 26 24 41	26
6	1000	6 15 16 17 18 20 32 25 26 24 41	23
7	35000	7 16 17 18 33 30 31 32 20 21 22 23 41	25
8	35000	8 18 33 30 31 32 25 22 42	23
9	35000	9 19 20 32 21 22 42	21
10	35000	10 31 32 25 22 42	20
11	140000	11 25 23 41	16
		11 27 26 24 41	17
12	140000	12 37 40	13
		12 24 41	15
13	140000	13 38 39	13
		13 35 36 37 24 41	18

objectives.

We tested our model on the Greater Houston (Texas) evacuation network and confirmed that our model can handle a very large-scale metropolitan evacuation network with little computation time. Sensitivity of the proposed evacuation plan to the number of evacuees and the number of evacuation paths was also analyzed. For the test network, a linear correlation was observed between the number of evacuees and the clearance time required for evacuation. Results indicated that the evacuation manager may need to compromise with the clearance time when the number of paths are fixed to a certain value less than the minimum number of paths required for a complete evacuation. Having more number of evacuation paths do not result in lowering of the clearance time.

In future, we plan to extend our work by developing stochastic evacuation models that can deal with random number of evacuees and random arc capacity that follow certain probability distributions.

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