Reliability Analysis of Evacuation Routes Under Capacity Uncertainty of Road Links

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Abstract

In this paper, we present a reliability based evacuation route planning model that seeks to find the congestion probability on a set of evacuation paths during evacuation. Most of the existing models for network evacuation assume deterministic capacity estimates for road links without taking into account the uncertainty in capacities induced by myriad external conditions. Only a handful of models exist in the literature that account for capacity uncertainty of road links. We extend the dynamic network based evacuation model and formulate a minimum cost network flow problem with probabilistic arc capacity constraints within the framework of chance constrained programming technique. We also use the concept of network breakdown minimization principle of traffic flow for evacuation planning problem and formulate a path based evacuation routing and scheduling model. Given the horizon time for evacuation, model selects the evacuation paths and finds flows on the selected paths that result in minimum congestion in the network and finds the reliability of the evacuation plan. Numerical examples are presented and we discuss the effectiveness of the stochastic models in evacuation planning. It is shown that the reliability based evacuation plan is conservative as compared to plans made using a deterministic model. Stochastic models guarantee that congestion can be avoided with a confidence level at the cost of increased clearance time.

Key words: Dynamic network flow problem, stochastic capacity, emergency evacuation, reliability, chance constrained programming.

1 Introduction

In many practically motivated decision problems a number of uncertain, unforeseen or not completely known factors may play a non-negligible role thus affecting the decisions taken without considering these factors. It is therefore advisable to explicitly consider such uncertainties during the planning phase. Traffic congestion is one such uncertainty that occurs under a great traffic demand during emergency evacuation of a geographic region. Occurrences of unexpected congestion will disturb the envisioned evacuation plan. For an illustration, consider the following real life example from Litman (2006): During Hurricane Rita the state's highway system in Houston, TX, became gridlocked and average travel time to reach Dallas was in the range of 24-36 hours, travel time to Austin was in the range of 12-18 hours and travel time to San Antonio was in the range of 10-16 hours, depending on the point of departure from Houston. These travel times are $4-8$ times higher than what would have been required in the free flowing traffic conditions.

The relation between travel time and roadway capacity can be best explained using the link performance function. According to Bureau of Public Records (BPR), the link performance function for average travel time as a function of flow volume and capacity is given by

$$t_a(\mathcal{Q}_a, \mathscr{U}_a) = t_a^f \left[1 + \beta \left(\frac{\mathcal{Q}_a}{\mathscr{U}_a}\right)^n\right], \tag{1}$$

where subscript a refers to a particular link in the set of links $\mathscr{A}$ with $t_a^f, \mathscr{U}_a$, and t_a, respectively, are link a's free-flow travel time (which is deterministic), capacity, and travel time with flow volume $\mathcal{Q}_a$; β and n are deterministic parameters associated with the BPR travel time function for which the value of $\beta = 0.15$ and $n = 4.0$ are typically used. Now consider a scenario where a fixed flow of vehicles is allocated to the link but the link capacity is subject to stochastic degradation (due to weather, accidents, driver behavior, etc.). In such a scenario, $\mathscr{U}_a$ is replaced by the random variable $\widetilde{\mathscr{U}_a}$. Then in (1), the link travel time t_a becomes a random variable $\tilde{t_a}$ with its mean and variance expressed as

$$E(\tilde{t_a}) = E\left(t_a^f\left[1 + \beta\left(\frac{\mathcal{Q}_a}{\widetilde{\mathscr{U}_a}}\right)^n\right]\right) = t_a^f + \beta t_a^f E\left[\left(\frac{\mathcal{Q}_a}{\widetilde{\mathscr{U}_a}}\right)^n\right], \tag{2}$$

$$var(\tilde{t_a}) = E[(\tilde{t_a})^2] - E^2(\tilde{t_a}). \tag{3}$$

Assuming that the free flow travel time t_a^f is deterministic and constant, expressions (2) and (3) allow the calculation of the expected value $E(\tilde{t_a})$ and variance $var(\tilde{t_a})$ of link travel time which depends on the probability distribution function of link capacity $\widetilde{\mathscr{U}_a}$. Using the arc transit time as

$$t_a = E(\tilde{t_a}) + var(\tilde{t_a}), \tag{4}$$

the total clearance time would, therefore, increase as compared to the scenario when the free flow speed is considered with deterministic capacity. This highlights the importance of accounting for congestion probability in order to make a realistic evacuation plan.

Definition 1. *Road capacity: The* Highway Capacity Manual *(2000) (HCM) defines capacity as "the maximum sustainable hourly flow rate at which persons or vehicles can reasonably be expected to traverse a point or uniform section of a lane or roadway during a given time period under prevailing roadway, environmental, traffic, and control conditions."*

Much of the evacuation planning literature considers the designed capacity of a roadway link as constant at all times. Maximum flow rate or traffic volume below this capacity is considered as acceptable and volume above this is unacceptable resulting in breakdown. However, it is well known from Chen et al. (2002), Lo and Tung (2003) and Persaud et al. (1998) that actual capacity of an arc representing a section of a road is not constant and is a function of volume of the vehicles present in that arc at a given time. In a realistic situation, the element of stochasticity which is locally a certain disorder in a queue of cars develops like a domino effect into a macroscopic phase transition from free to congested flow. Mahnke et al. (2005) describe this phenomenon as *traffic breakdown*. According to Kerner's three-phase traffic theory (Kerner (2011)) (see Figure 1), when the flow rate q of the link exceeds a certain threshold value q_{th}, then network enters a metastable state where a traffic breakdown occurs with some finite probability and this probability approaches a value of 1 when the flow exceeds the maximum volume q_{max} possible for the link. Existing evacuation planning models with an objective of minimizing the clearance time pushes maximum flow out of the network which is limited only by the arc capacity and do not consider congestion probability.

Stochastically degrading capacity of the road link would result in congestion which subsequently would increase the clearance time for the network. Thus, not accounting for capacity uncertainty and congestion probability may lead to suboptimal or possible infeasible solutions in real evacuation situations. This calls for a new mathematical approach that incorporates the uncertainty inherent in the estimates of the roadway capacity to come up with a reliable evacuation plan.

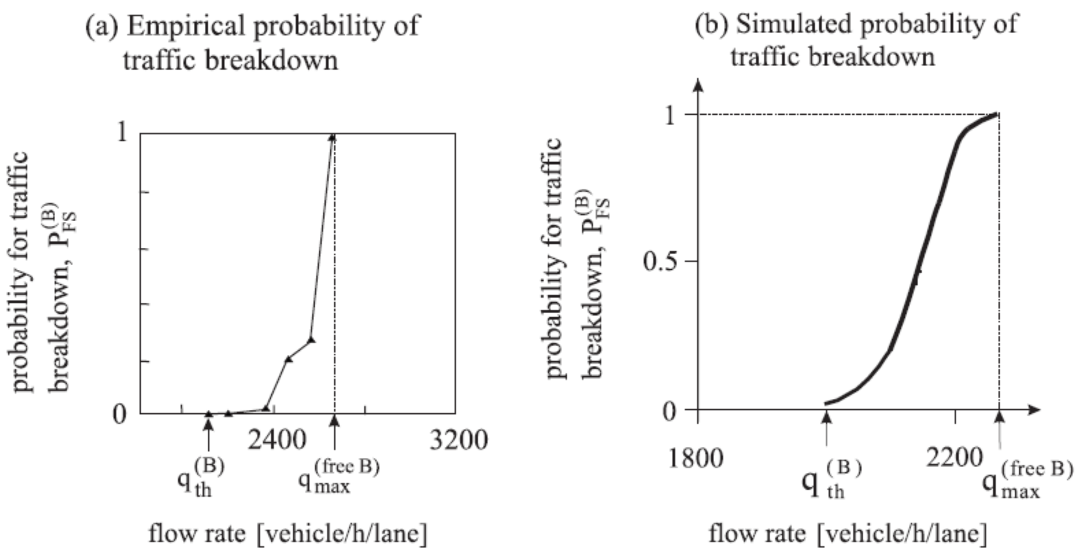

Figure 1: Probability of traffic breakdown (Kerner (2011))

The literature on evacuation planning is rich and a plethora of models have been proposed to handle different versions of the problem. A comprehensive review can be found in the papers by Aronson (1989), Hamacher and Tjandra (2002) and Yusoff et al. (2008). Optimization and control theories of vehicular traffic are usually based on the minimization of travel time by utilizing the maximum roadway capacity. Evacuation planning literature for short notice evacuation planning considers time as the primary parameter and seeks to find the paths and flow that minimizes the evacuation time. Chiu et al. (2007) proposed a dynamic cell transmission based model for no-notice evacuation. Researchers have also designed models that aims at efficiently utilizing the limited transportation infrastructure. Cova and Johnson (2003) proposed a mixed-integer linear programming model for lane-based routing to avoid conflicts. Contraflow research by Lu et al. (2005) and Kim and Shekhar (2005) aims to increase capacity of the network by opening up the reverse lanes for evacuation flow. The problem with added complication of priority based evacuation routing was addressed by Liu et al. (2006). A mixed integer path based network flow model was recently developed by Lim et al. (2012) that finds evacuation paths and schedules for vehicular flow. Lim and Reza Baharnemati (2010) proposed solving the evacuation problem using path based model as it is easier for determining the evacuation routes and traffic flow as opposed to the arc based model. In the arc based model, the number of variables becomes very large for practical evacuation networks and is thus not easy to solve. Rungta et al. (2012) proposed a framework for simultaneous minimization of clearance time and evacuation routes using a similar dynamic path based network flow model. Overall, this set of literature addresses many sub-problems within the evacuation planning umbrella via deterministic models.

In contrast to deterministic models, the literature dealing with uncertain and dynamic conditions inherent in emergency evacuation is relatively sparse and has only recently been given due importance by the research community. Choi et al. (1988) first addressed the building evacuation problem with arc capacity as a function of flow in incident arcs. For regional evacuation, Yazici and Ozbay (2010) used chance constrained programming to model uncertainties in road capacities and demand origination during evacuation. Travis Waller and Ziliaskopoulos (2006) developed a demand uncertainty model and later a model was proposed by Ng and Waller (2010) that gave the probabilistic guarantees on the evacuation plan considering uncertainty in the number of evacuees and arc capacities. Their work proposed demand inflation and capacity reduction necessary to ensure a certain reliability level. Ng and Waller (2011) extend their work on capacity uncertainty by considering symmetric probability distributions for the random capacity and provided the reliability bound for a stochastic user equilibrium model. Robust extension of evacuation problem has been considered in the works by Yao et al. (2009), Chung et al. (2011), and Ben-Tal et al. (2011) where they use the paradigm of uncertainty sets and develop a robust tractable model to address the demand uncertainty. Miller-Hooks and Sorrel (2008) proposed a noisy genetic algorithm to find the maximum expected number of evacuees who can successfully evacuate within a given egress time considering variable time and roadway capacity with known distribution functions. Stepanov and Smith (2009) has approached the stochastic evacuation as a queuing model to avoid congestion. None of the work in the evacuation literature, however, considers capacity in the context of traffic breakdown and model the problem with objective of minimizing the probability of congestion. Minimizing clearance time which is central to all the models is very much dependent on the hypothesis of fixed transit time on the arcs and the calculations can be misleading in case of the traffic jam buildup. Therefore, our study aims to model the mass evacuation with stochastic arc capacity having an objective of minimizing the network congestion.

Traffic routing and scheduling problems usually rely on either Wardrop's user equilibrium *(UE)* or system optimum *(SO)* traffic flow principles proposed by Wardrop (1952) to determine the clearance time in current evacuation literatures. *SO* principle is the preference of the evacuation managers trying to minimize the network-wide travel time and *UE* principle reflects the wish of the drivers to reach their destinations as soon as possible. During evacuation, how the situation will progress is uncertain and a traffic breakdown occurs in the network with some probability which is not taken into account by the network travel cost optimization principles of Wardrop. In this paper, we use network breakdown minimization *(BM)* principle proposed by Kerner (2011). *BM* principle for traffic assignment aims for assigning link flow rates that minimize the probability of traffic breakdown in a network.

Providing a reliable flow is important in the context of emergency evacuation. *A priori* analyses of envisioned evacuation paths for traffic reliability with high probability would guarantee that actual evacuation does not result in undesirable surprises. This paper, therefore, lays emphasis on finding an evacuation plan considering variable arc capacity with known distribution function which would result in a free flowing traffic without any congestion. Our overall strategy to address capacity uncertainty and congestion minimization in the network evacuation problem is to use the capacity distribution function and thus find the traffic reliability estimate. The key contributions of this paper are as follows: a) we model an optimization problem that minimizes the probability of congestion in a stochastic network setting; b) we find a relationship between the clearance time, number of evacuation paths and congestion probability; and c) our approach acts as a verification model for the evacuation plan and reports the reliability in terms of confidence level with which a congestion might occur in the network if the said plan is used.

The rest of the paper is structured as follows. Section 2 introduces the modeling approach for uncertain arc capacity within the framework of chance constrained programming. We propose a model for finding the evacuation routes and traffic flow that will result in minimum congestion in the network. The stochastic model under mathematical optimization framework is described. Section 3 reports the solution approach and computational results. Finally, conclusions and future research directions are discussed in Section 4.

2 Evacuation Planning under Uncertain Arc Capacity

In this section, we first describe the deterministic minimum cost network flow problem for finding the minimum clearance time. Subsequently, we introduce the model with random capacity of arcs and find the minimum clearance time for a desired reliability level. Further, we propose a path based model for finding the path reliability and determining a reliable flow on evacuation paths to be used between each origin-destination (O-D) pair.

2.1 Deterministic Minimum Cost Flow Evacuation Problem

A time expanded network flow model has been used to mathematically represent traffic flow evolution in an evacuation network for this optimization model. The network consists of a graph with capacities and transit times associated with the arcs. Consider a directed static network $\mathscr{D} = (\mathscr{N}, \mathscr{A})$ with $\mathscr{N}$ and $\mathscr{A}$ as the set of nodes and arcs, respectively. The time expansion of $\mathscr{D}$ over a time horizon T defines the dynamic network $\mathscr{D}_T = (\mathscr{N}_T, \mathscr{A}_T)$ associated with $\mathscr{D}$ having holdover arcs and movement arcs. Holdover arcs are virtual road sections represented on a time expanded network whose content represents the number of vehicles still remaining at the source node $\mathscr{N}_c$ and the capacity of these arcs are equal to the capacity of the source nodes. Movement arcs represent the actual road link of a traffic network at different time interval and its content represents the movements of vehicles from one node to another. The flow on the movement arcs are limited by their maximum capacity $\mathscr{U}_{ij}$. It is this capacity of movement arcs that we consider as random in the stochastic model.

We denote as $\mathbb{T}$, the set of discrete time intervals, i.e., $\mathbb{T} = \{0, 1, \ldots, T-1\}$. The time expansion essentially is the replication of the static network $\mathscr{D}$ at each discrete unit of time in $\mathbb{T}$. Since there are multiple copies of sink nodes, a super sink node $\mathscr{N}_d^+$ is introduced to create a single sink network. In the dynamic network flow model, the flow variable y_{ij}^t is the number of vehicles that leave node i at time t and reach node j at time $t + \sigma_{ij}$ where σ_{ij} represents the transit time on arc (i, j). The primary goal of the model is to find the lower bound of the clearance time for the underlying network with a given initial supply of vehicles. Therefore, we wish the network to have nodes separated by unit transit time such that it is possible to capture the precise time till there is flow in the network. We modify the network such that $\sigma_{ij} = 1$ for all the arcs in the modified network by introducing dummy nodes between the nodes having travel time greater than 1. Flow variable x_i^t represents the number of vehicles that move on holdover arcs. Let $\mathscr{N}_s$ denote the set of destination nodes, $\mathscr{C}_j$ as the total capacity of the destination node j and $\mathcal{S}_i$ as the initial demand at source node i. Arc (i, j) is also alternatively represented as arc a in this paper. Shown below is the deterministic model for finding the minimum clearance time. This is a minimum cost flow model and we name the model as MET-D.

$$\text{Minimize:} \quad \sum_{(i,j)\in\mathscr{A}(\mathscr{N}_s)} \sum_{t\in\mathbb{T}} t \cdot y_{ij}^t \qquad (MET-D) \tag{5}$$

$$\text{Subject to:} \quad x_i^0 + \sum_{(i,j)\in\mathscr{A}(i)} y_{ij}^0 = \mathcal{S}_i \qquad \forall i \in \mathscr{N}, \tag{6}$$

$$x_i^t - x_i^{t-1} + \sum_{(i,j)\in\mathscr{A}(i)} y_{ij}^t - \sum_{(j,i)\in\mathscr{A}^{-1}(i)} y_{ji}^{t-1} = 0 \qquad \forall t \in \mathbb{T}\setminus\{0\}, \quad \forall i \in \mathscr{N}, \tag{7}$$

$$x_i^t - x_i^{t-1} - \sum_{(j,i)\in\mathscr{A}^{-1}(i)} y_{ji}^t = 0 \qquad \forall t \in \mathbb{T}, \quad i = \mathscr{N}_d^+, \tag{8}$$

$$\sum_{(j,i)\in\mathscr{A}^{-1}(\mathscr{N}_d^+)} \sum_{t\in\mathbb{T}} y_{ji}^t = \sum_{i\in\mathscr{N}_c} \mathcal{S}_i, \tag{9}$$

$$\sum_{t\in\mathbb{T}\setminus\{0\}} \sum_{(j,i)\in\mathscr{A}^{-1}(i)} y_{ji}^{t-1} \leq \mathscr{C}_i \qquad \forall i \in \mathscr{N}_s, \tag{10}$$

$$y_{ij}^t \leq \mathscr{U}_{ij} \qquad \forall t \in \mathbb{T}, \quad \forall (i,j) \in \mathscr{A}, \tag{11}$$

$$x_i^{|T|-1} = 0 \qquad \forall i \in \mathscr{N}, \tag{12}$$

$$0 \leq x_i^t \leq \mathscr{C}_i \qquad \forall t \in \mathbb{T}, \quad \forall i \in \mathscr{N}, \tag{13}$$

$$y_{ij}^t \in \mathbb{Z}^+, x_i^t \in \mathbb{Z}^+. \tag{14}$$

As in any network model, the movements of vehicles between nodes is defined by flow propagation and flow conservation equations (6) - (8). These relations decide respectively the flows y_{ij}^t between two nodes based on upstream/downstream traffic flow on the arcs and depict the evolution of the node status (i.e., the number of vehicles in each node x_i^t) over time. Constraint (9) states that the total incoming flow into the super-sink node $\mathscr{N}_d^+$ should be equal to the total supply at the start of the analysis period. The implication of this constraint is that it does not allow any withholding at the impact nodes and thereby pushing for the complete evacuation of the network. It pushes the flow towards the sink nodes which are connected to the super-sink node and thus result in flow propagation in the network. The total amount of flow, however, is determined by the objective function (5). Constraint (10) specifies that the total incoming flow into the set of sink nodes $i \in \mathscr{N}_s$ should not exceed the capacity of the sink nodes. A limiting constraint on the maximum flow possible on any arc in the arc set $(i, j) \in \mathscr{A}$ at any time $t \in \mathbb{T}$ is expressed in constraint (11). Constraint (12) specifies the model to push for zero vehicles that are left behind at the end of the analysis period. Constraint (13) limits the node capacity x_i^t to its maximum capacity $\mathscr{C}_i$. It should be noted that the initial assignment period T should be high such that all traffic assigned in the network exits the network, otherwise they would be left behind and problem will not meet the constraints.

In this model, a deterministic capacity estimate of the arcs is used to limit the flow on paths. This model results in a minimum clearance time estimate along with the flows on the arcs as per the deterministic capacity. Deterministic capacity as mentioned in HCM (2000) is the maximum sustainable hourly flow rate that can be achieved repeatedly during peak periods. But the demand volume that causes breakdown varies in real traffic flow and the flow rate observed during a breakdown depends on the behavior of drivers thus making the arc capacity a random variable $\widetilde{\mathscr{U}_{ij}}$. Congestion occurs at an arc (i, j) with a finite probability p when the optimal flow y_{ij} obtained from the deterministic model exceeds the realized capacity.

When the demand on an arc exceeds its capacity, or capacity decreases to a level less than demand, then congestion occurs resulting in a bottleneck. Therefore, the clearance time calculated using deterministic capacity may not be enough to evacuate using the selected evacuation paths and the corresponding flow rate. In such situations of stochastically degrading capacity, the clearance time calculated using the deterministic model would result in an infeasible evacuation plan. Given the consequences of the deterministic capacity, our next task is to model the problem as a probabilistic constrained program and to incorporate a reliability measure for the variable arc capacity of the links.

2.2 Chance Constrained Model

We consider a probabilistic programming approach to model the evacuation problem assuming that the probability distribution of the random arc capacity is known. Inability of the traffic flow on a road section to meet the capacity requirement of that arc at all times is modeled using chance constrained programming. The probability level is set to a value which ensures that the flow is assigned in such a way as to meet criteria most of the time. More specifically, we find the deterministic equivalent of the variable capacity that should be used in the model such that the probability with which the flow value might exceed the capacity is bounded within a pre-specified tolerance level. This can be thought of as determining of the *a priori* traffic flow on the arcs where the future capacity variations are already accounted for.

Referring to the deterministic model, the arc capacity constraint (11) is modified to ensure the feasibility of capacity constraint for each arc within a reliability level ε_{ij}. The modified constraint is of the form

$$\Pr\left(y_{ij}^t \leq \widetilde{\mathscr{U}_{ij}}\right) \geq \varepsilon_{i,j}, \quad \forall (i,j) \in \mathscr{A}. \tag{15}$$

Keeping other constraints of the MET-D model same, the model with the modified arc capacity constraint is termed as MET-S. Constraint (15) is the individual chance constraint equivalent of the deterministic constraint (11) with the desired probability level imposed individually on each constraint. Parameter $\varepsilon_{ij} \in (0, 1]$ is the desired reliability level and the value of ε is set such that the optimal solution to the approximation of the chance constraint is feasible to the probabilistic constrained programming (PCP) model. Capacity uncertainty is denoted by random variable $\widetilde{\mathscr{U}_{ij}}$ and we assume that the distribution function of the random capacity is known.

Consider the cumulative distribution function (CDF) for capacity in terms of probability when the traffic volume y_{ij} on the link exceeds its capacity $\mathscr{U}_{ij}$, i.e.,

$$F_{\mathscr{U}_{ij}}(y_{ij}) = p(\mathscr{U}_{ij} \leq y_{ij}). \tag{16}$$

This implies that the overload probability for a single bottleneck is equal to the CDF of capacity. The probability of no congestion, i.e., $p(\mathscr{U}_{ij} > y_{ij})$, can be expressed as the complementary event:

$$P_{\mathscr{U}_{ij}}(y_{ij}) = 1 - F_{\mathscr{U}_{ij}}(y_{ij}). \tag{17}$$

Considering traffic breakdown as a failure event to estimate the capacity $\mathscr{U}_a$ of an arc, Brilon et al. (2005) empirically concluded that when the flow rate exceeds a certain threshold value $\mathscr{Q}_a$ then there is a finite probability with which traffic volume can exceed the capacity of the link and congestion can occur. We define this saturation flow rate as the capacity of the arc. After reaching a certain upper bound of flow rate $\mathscr{Q}_{a_{max}}$ for a particular link, this probability reaches unity and it is certain that congestion would occur in that link. From his work, CDF for link capacity was found to follow *Weibull* distribution, i.e.,

$$F_{\mathscr{U}_a}(\mathscr{Q}_a) = 1 - e^{-(\frac{\mathscr{Q}_a}{\beta_a})^{\alpha}}, \tag{18}$$

where α and β are respectively, shape parameter and scale parameter. Shape parameter α is typically having a value in the range of $9 - 15$ for a three lane road. Weibull distribution is used in this paper to model the uncertainty of link capacity with stochastic traffic volume $\mathscr{Q}_a$ used as a the link capacity constraint. Other discrete or continuous distributions can also be used that better approximates the random capacity of a roadway section. For example, Siu and Lo (2008) consider road capacity following a general uniform distribution in their study.

Probabilistic constrained models are usually solved using a deterministic approximation. The difficulty in solving such models arises mainly from the fact that the chance-constraint set (15) may not be convex (Nemirovski and Shapiro (2007)). Proving the convexity of the chance-constraint set (15) would thus simplify the optimization of the problem by suitable approximations of the chance-constrained function.

Corollary 2.1. *The feasible region of chance constraint* (15) *is convex.*
Proof: *Constraint* (15) *is of the form* $\Pr\{Ax \leq \tilde{b}\} \geq p$*, where,* A *is a deterministic coefficient matrix and* $p \in (0, 1]$ *is given. Variable capacity vector* $\tilde{b}$ *follows Weibull distribution and for* $\alpha > 1$*, it is a log-concave distribution. In this case, according to Prékopa (1970), chance constraint feasible set* (15) *is convex.*

The individual chance constraint (15) of the form $\Pr\{Ax \leq \tilde{b}\} \geq p$ has a separable structure with random right hand side (RHS). Such probabilistic constraints can be solved using a deterministic approximation by replacing the values of random RHS with the corresponding p^{th} quantile of the distribution function. This is effectively translating the assumption on capacity uncertainty into equivalent conservative deterministic saturation flow rate that would ensure the constraint feasibility with probability ε_{ij}. More specifically, the probabilistic constraint can be re-written in its deterministic equivalent as

$$y_{ij}^t \leq P_{\mathscr{U}_{ij}}^{-1}(\varepsilon_{ij}), \tag{19}$$

where the RHS of the above equation is rounded to the nearest integer value to ensure that the capacity is an integer. The model can then be solved after substituting the deterministic equivalent of probabilistic constraint for each arc (Charnes and Cooper (1959)).

Proposition 1. *The optimal objective value of the approximation of PCP upper bounds the optimal objective value of PCP.*
Proof: *Consider the following situation.*

$$P_{\mathscr{U}_{ij}}^{-1}(\varepsilon_{ij}) \leq \widetilde{\mathscr{U}_{ij}} \tag{20}$$

Finding the nominal flow value based on the deterministic assumption of random capacity for the above mentioned scenario might result in under-utilization of the roadway capacity, thereby, resulting in upper bound for the objective value.

According to Proposition (1), solutions obtained using the deterministic equivalent of the probabilistic constraint are conservative. The objective value corresponding to the minimum clearance time obtained using the PCP model could be higher than the optimal objective value if everything was known and deterministic. But, the solution is certainly more reliable in the wake of uncertain arc parameters.

2.3 Minimum Congestion Path

The primary aim of this paper is to address tactical preparedness concerns involving the flow on the paths and clearance time by minimizing the traffic breakdown which might occur at bottlenecks and carefully plan for evacuation considering the stochastically degrading arc capacity. Section 2.2 was devoted to find the clearance time where the reliability was set at the link level. For finding the reliability at the route level, we propose a path based model that is designed to find paths with a minimum probability level of congestion for the complete network. As opposed to arc based model, the path based model is used because it reduces the problem complexity. Moreover, the path based model finds out the evacuation routes and starting schedules for the vehicle loading in the network. The proposed path based model in this paper is based on the principle of bottleneck minimization (*BM*) proposed by Kerner (2011). We apply *BM* principle in a static network setting to incorporate the discrete uncertainty of link capacity in our model within the mathematical programming framework. Unlike other works in evacuation literature (Aronson (1989), Chiu et al. (2007), Hoppe and Tardos (1994)) that use Wardrop's *SO* or *UE* principle, we prefer *BM* principle so that congestion minimization is given a priority.

For the path based model, the underlying assumption is that a list of paths between each O-D pair is known *a priori*. Path enumeration for the evacuation network can be achieved using a shortest path model for finding a pool of unique paths between each O-D pair. Given the pool of paths, a model is designed to select evacuation routes, evaluate their reliability and find an evacuation plan that will result in minimum congestion in the network. The objective is to minimize the maximum probability of congestion that might occur in the evacuation paths during evacuation of a given number of evacuees within the time bound T. Although, the expected results obtained using this principle would be highly conservative, it can give the emergency personnel a reasonable probabilistic guarantee for smooth execution of the plan without any undue surprises.

Before moving to the formulation, we define the probability of free flow of vehicles in terms of the CDF of capacity function of the arcs constituting the path. Since a path is a sequence of arcs in series, the distribution function associated with the path would be the product of individual distribution function of each arc contributing to the path. Here, we assume that each arc is independent and do not affect other arcs. Accordingly, the free flow probability which is the compliment of congestion probability of the path can be stated as

$$P_{free}(\mathcal{Q}_1, \mathcal{Q}_2, \ldots, \mathcal{Q}_n) = \prod_{i=1}^{n} [1 - F_{\mathscr{U},i}(\mathcal{Q}_i)] = e^{-\sum_{i=1}^{n} (\frac{\mathcal{Q}_i}{\beta_i})^{\alpha}}. \tag{21}$$

The last equality follows from the assumption that the CDF of capacity follows a Weibull distribution. If the capacity is assumed to follow some other distribution, then the equation can be modified accordingly.

The problem is first modeled as a chance constrained problem and we call this model as MCP-J. Notations specific to the model are integer decision variables f_p that represent the flow on path $p \in \mathscr{P}$ and binary decision variables y_p which denotes the decision for path selection in the final evacuation plan. Real variable $\gamma \in (0, 1]$ represents the probability of congestion in the network. Parameter σ_p represents the travel time of path $p \in \mathscr{P}$ and sets $\mathbb{O}_p$ and $\mathbb{D}_p$ represent respectively the set of source and sink nodes. Assuming that the evacuation time T is known, the mini-max model to decide the minimum number of paths and minimize the maximum congestion among the paths can be stated as

$$\text{Minimize} \quad Z = \sum_{p \in \mathscr{P}} y_p + \gamma \qquad (MCP - J) \tag{22}$$

$$\text{Subject to:} \quad \Pr\left\{ \bigcap_{a \in p} \left(\sum_{p' \in \mathscr{P}} f_{p'} \cdot \delta_{p'a} \leq \tilde{\mathscr{U}}_a \right) \right\} \geq 1 - \gamma, \qquad \forall p \in \mathscr{P}; \tag{23}$$

$$\sum_{p|\mathbb{O}_p = i} (T - \sigma_p) f_p \geq \mathcal{S}_i, \qquad \forall i \in \mathscr{N}_c; \tag{24}$$

$$\sum_{p|\mathbb{D}_p = j} (T - \sigma_p) f_p \leq \mathscr{C}_j, \qquad \forall j \in \mathscr{N}_s; \tag{25}$$

$$(T - \sigma_p) f_p \leq M \cdot y_p, \qquad \forall p \in \mathscr{P}; \tag{26}$$

$$f_p \in \mathbb{Z}^+, \mathscr{U}_a \in \mathbb{Z}^+, y_p \in \{0, 1\}, \gamma \in (0, 1] \qquad \forall p \in \mathscr{P}, \forall a \in \mathscr{A}; \tag{27}$$

The deterministic model without considering stochastic arc capacity is given in Appendix. In model MCP-J, constraint (23) is an individual chance constraint which states that for each arc $a \in p$, the summation of flow on the arc over all the selected paths $p' \in \mathscr{P}$ is less than a random capacity with a probability of free-flow greater than variable $(1 - \gamma)$. We use the joint probability distribution for the capacity of arc in path and is given in equation (21). This CDF is again a Weibull distribution having log-concave property. According to Corollary 1, a feasible set of chance constraint (23) is, therefore, convex and we can use a deterministic approximation of the capacity to solve the model. For solving MCP, we reformulate the model into a deterministic model which is a mixed integer non-linear program (MINLP). Specifically, constraint (23) is replaced by the following constraints in the MINLP model.

$$\sum_{p \in \mathscr{P}} f_p \cdot \delta_{pa} \leq \mathcal{Q}_a, \quad \forall a \in \mathscr{A}; \tag{28}$$

$$e^{-\sum_{a \in p} \left(\frac{\mathcal{Q}_a}{\beta_a}\right)^{\alpha}} \geq 1 - \gamma, \quad \forall p \in \mathscr{P}; \tag{29}$$

In the formulation, the uncertainty only affects the capacity vector $\mathscr{U}_a$ of arcs in arc capacity constraint (23). We define $\mathcal{Q}_a$ as an auxiliary variable whose value is determined according to the probability level $1 - \gamma$ which is in turn determined by the objective function. The only restriction on the uncertainty set $\mathcal{Q}_a$ is that $\mathcal{Q}_a \subseteq \mathbb{Z}_+^n$ (this corresponds to the requirement of non-negative integer capacities). For each arc $a \in \mathscr{A}$, the arc attribute values are random variables and are specified at the entrance to an arc. They are assumed to be static for that particular traveler until exiting the arc. This property is referred to as frozen link property by Orda and Rom (1990). Capacity distribution functions for each arc are statistically independent assuming that arc lengths are sufficiently large. It can be assumed that the roadway capacity does not exhibit significant fluctuations for small road segments (Yazici and Ozbay (2010)). Therefore, the realization of the network is spatially independent. Everything else in the constraint matrices are assumed certain.

Congestion probability level γ and auxiliary variable $\mathscr{Q}_a$ for each arc are interdependent and are determined such that the objective function is minimized. Note that the model pushes for the complete evacuation within time bound T which is mathematically represented in constraint (24). Constraint (25) ensures that the capacity of the sink nodes is not exceeded at all times. Constraint (26) is the path selection constraint from each source node. The optimal value of objective function (22) can be obtained by choosing the paths that minimize the probability of maximum congestion among all the selected paths. Here paths are selected from a pool of possible evacuation paths set $\mathscr{P}$ given as an input to the model.

Path selection is a combinatorial problem and the model selects the best set of paths that can completely empty the network in time T. According to the definition of the traffic reliability, in case of overload of one section, the whole system is considered as overloaded. Therefore, for each network state, the objective function minimizes the maximum probability of congestion among all the paths. The solution will provide the reliability of the chosen paths to be used for evacuating the given number of evacuees within time T in terms of the congestion probability γ. A flow pattern for the paths corresponding to the resulting reliability is also obtained. This flow would be employed on the assumption that network state is not known to the evacuees at the time of evacuation.

Probabilistic arc capacity constraint is also formulated as an individual chance constraint with probability level assigned to each arc separately. The constraint would look like

$$\Pr\left\{\sum_{p\in\mathscr{P}} f_p \cdot \delta_{pa} \leq \tilde{\mathscr{U}}_a\right\} \geq 1-\gamma_a, \quad \forall a \in \mathscr{A}; \tag{30}$$

In such models, the CDF of individual arcs is used to find the deterministic estimate. This model which we term as MCP-I would yield a solution that binds the probability constraint at each arc. On the other hand, the individual chance constrained model MCP-J would result in a solution with a network wide reliability and the probability constraint is bounded for each path.

Theorem 2.2. *MCP-I provides a tighter bound as compared to MCP-J*
Proof: *Let n be the number of arcs in the path. Using Bonferroni's inequality, the following inequality holds true.*

$$\Pr\left\{\bigcap_{a\in p}\left(\sum_{p'\in\mathscr{P}} f_{p'} \cdot \delta_{p'a} \leq \tilde{\mathscr{U}}_a\right)\right\} \geq 1-n+\sum_{a\in p}\Pr\left\{\left(\sum_{p'\in\mathscr{P}} f_{p'} \cdot \delta_{p'a} \leq \tilde{\mathscr{U}}_a\right)\right\} \tag{31}$$

i.e., probability of free flow along the path is greater than the probability of free flow when calculated individually along the arcs on the path.

Using Theorem (2.2), it can be concluded that the solution of MCP-I would be more conservative as compared to solution obtained using model MCP-J. This simply means that the congestion probability γ would be smaller for MCP-I model but at the cost of conservative flow on the arcs and increase in clearance time for the network.

3 Computational Results

In this section, we report numerical results from the solution obtained for the proposed models. Models are solved on a 3.07 GHz workstation with 24 GB of memory running on Ubuntu 10.04.3 operating system.

Before undertaking an extensive set of computational results, we now put forth the following questions we wish to answer:

1. What is the probability of congestion for a deterministic evacuation route plan?

2. How different is the stochastic schedule compared to the deterministic counterpart?

3. How does the stochastic model assist in decision making?

Two numerical examples are provided for the illustration of the proposed models and to answer the questions posed. Experimental studies are conducted on the evacuation network of the Greater Houston Metropolitan area shown in Figure 2. The first experiment studies the impact of stochastic capacity on the clearance time and shows the variation of clearance time according to the desired reliability level. For the second experiment, we develop an evacuation plan that yields most reliable paths that should be used during evacuation with a random arc capacity.

3.1 Random Capacity and Bound on Clearance Time

In this section the effect of stochastic capacity on the clearance time is investigated. Stochastic programs are solved by using the deterministic approximations of the random variables in the model and the results are interpreted within a prescribed probability level on the probabilistic constraint. The evacuation network topology as shown in Figure 2 has multiple sources and destinations. A super-sink destination is connected to this network from all the destination nodes with arcs having infinite capacity. There are 42 nodes of which 13 are the source nodes, 4 destination nodes and the remaining nodes are intermediate nodes. Assuming that there are a total of $56,600$ evacuating vehicles present at the source nodes, analysis is done for finding the clearance time. Clearance time is first calculated assuming that arc capacities are deterministic and constant. Next, stochastically degrading capacities are considered and clearance time is found based on the desired reliability level.

For our calculation, an average value of $\alpha = 12$ is used for the shape parameter of Weibull distribution. Scale parameter β of the Weibull distribution varies as a function of different geometric and control conditions, different driver and vehicle populations and prevailing travel purposes. For an illustration purpose, we assume only two kinds of arcs for this network and the β values are set as either 52 or 104 when the maximum arc capacity is assumed to be 50 or 100, respectively. The MIP model MET-D is solved using CPLEX solver and the lower bound for the clearance time considering deterministic model was found to be $T = 128$. Results for different reliability level for the stochastic model MET-S is shown in Figure 3.

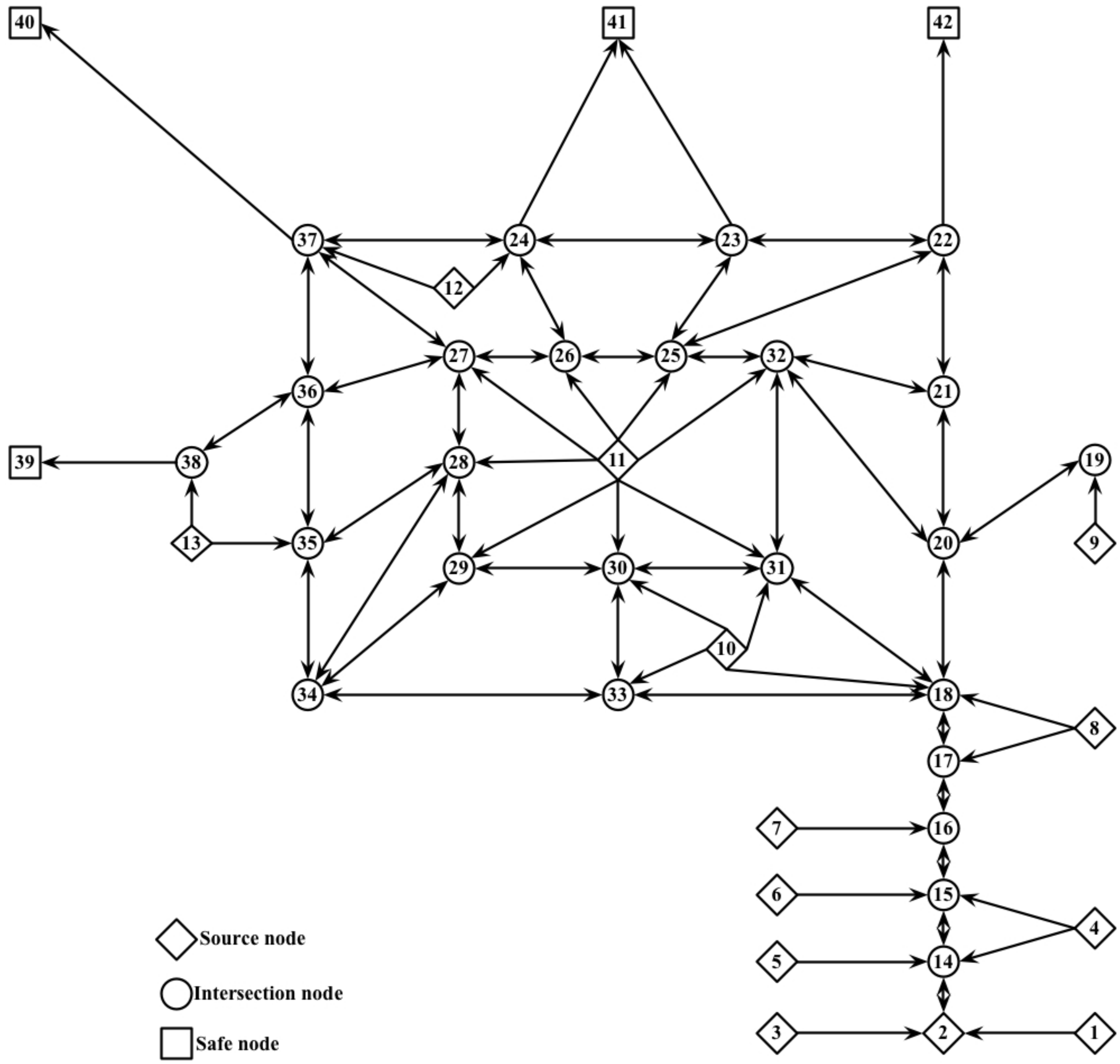

Figure 2: Representation of Houston evacuation network

It can be seen that a higher reliability level results in increasing of the clearance time. By planning for a decreased capacity, the evacuation plan becomes more reliable in the sense that the probability of deviating from this plan due to degrading capacity of the road link gets smaller. This essentially shows that to ensure the desired reliability level, the network is loaded in a conservative manner so as to avoid future infeasibility of the arc capacity constraint when the flow volume might exceed the capacity and result in congestion. Planning for extreme reliability would theoretically ensure that the deviation from the resulting evacuation plan would be impossible. A judgmental decision has to be taken regarding the desired reliability level of the evacuation plan.

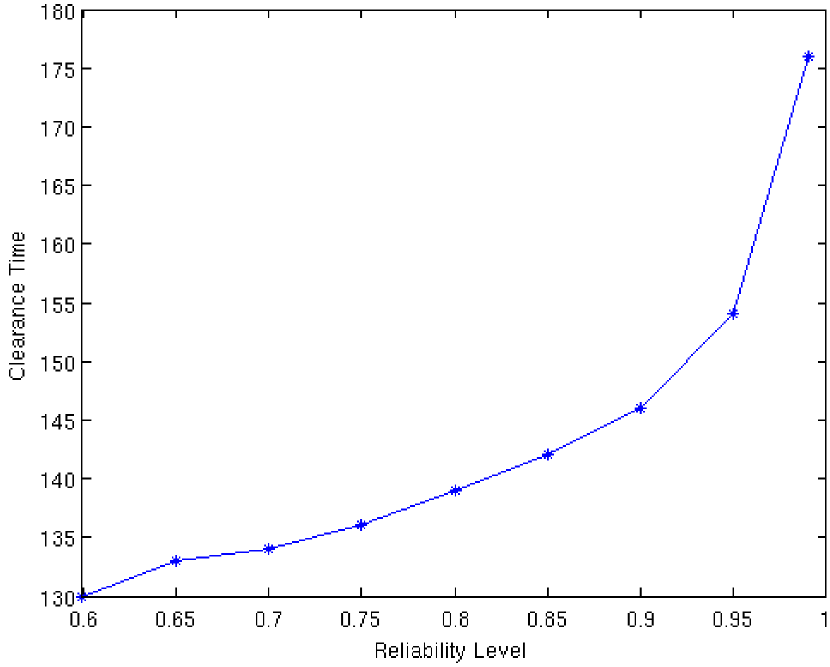

Figure 3: Clearance time for various values of reliability level

3.2 Evacuation plan with minimum congestion

We formulated an optimization model for finding an evacuation plan with minimum probability of congestion. This model would give the evacuation paths along with a fixed flow on the selected paths to be used to evacuate the vehicles from the region within time T given as an input to the model. First, consider the case when there is no uncertainty. The objective of deterministic MCP model (see Appendix) is to find the minimum number of evacuation paths. Given the clearance time T as an input, the model finds the path and the flow associated with the paths for complete evacuation. In the deterministic model, all the parameters are assumed constant and the model can be easily solved using commercially available solvers such as CPLEX. A comprehensive set of paths are provided to the model as an input parameter. MCP is a combinatorial optimization problem and an evacuation plan obtained using this deterministic model is shown in Table 1. This table shows that a total of 16 selected paths along with the corresponding flow that would ensure a complete evacuation within $T = 130$.

Now consider the scenario that there is an uncertainty associated with the capacity of arcs. In such a case, one might attempt to re-solve the evacuation model with new estimates of arc capacities in real-time. However, this might not be computationally feasible for large-scale networks. Even if this is feasible, there might not be enough time to communicate the new information to the evacuees and the emergency personnels executing the plan. There is a need to incorporate capacity uncertainty in the model so that the obtained evacuation plan can be followed without modifications with a desired confidence level. Probabilistic road capacity formulation MCP-J is used in such

Table 1: Evacuation plan using the deterministic model

Source Node	Total Vehicles	Selected Path between O-D pair	Travel Time	Flow
1	100	1 2 14 15 16 17 18 31 32 25 23 41	24	1
2	100	2 14 15 16 17 18 33 30 31 32 25 22 42	27	1
3	100	3 2 14 15 16 17 18 31 32 25 23 41	24	1
4	100	4 15 16 17 18 31 32 21 22 23 41	23	1
5	100	5 14 15 16 17 18 20 21 22 42	24	1
6	100	6 15 16 17 18 20 21 22 42	23	1
7	3500	7 16 17 18 20 21 22 42	22	33
8	3500	8 18 33 34 29 28 27 36 37 40	20	32
9	3500	9 19 20 21 22 42	20	32
10	3500	10 31 32 25 22 42	20	32
11	14000	11 27 37 40	14	26
		11 25 23 41	16	97
12	14000	12 37 40	14	22
		12 24 41	15	100
13	14000	13 38 39	13	100
		13 35 36 37 40	15	20

scenarios to find the evacuation plan.

MCP-J has dual objectives that find evacuation paths and their corresponding flows by minimizing congestion within a given clearance time T. Values of Weibull parameters α and β are assumed to be same as in the last section. Distribution function of arc capacity with the typical value of α makes the MCP-J model highly non-linear. Moreover, the integral limitations for the decision variables and the combinatorial nature of the problem because of the presence of 0-1 variables make this MINLP problem computationally intractable, especially for large scale instances with more than hundreds of variables. We follow the following algorithm to solve the model.

1. Relax the binary variable and solve the resulting Non Linear Programming (NLP) subproblem of the MINLP. If $y^{(0)} = y$ is integer, stop("integer optimum found"). Else goto step 2.

2. Find an integer point $y^{(1)}$ with a Mixed Integer Program (MIP) master problem that features an augmented penalty function to find the minimum over the convex hull determined by the half-spaces at the solution $(x^{(0)}, y^{(0)})$.

3. Fix the binary variables $y = y^{(1)}$ and solve the resulting NLP. Let $(x^{(1)}, y^{(1)})$ be the corresponding solution.

4. Find an integer solution $y^{(2)}$ with a MIP master problem that corresponds to the minimization over the intersection of the convex hulls described by the half-spaces of the KKT points at $y^{(0)}$ and $y^{(1)}$.

5. Repeat steps 3 and 4 until NLP subproblems start worsening (i.e., the current NLP subproblem has an optimal objective function that is worse than the previous NLP subproblem).

Note that a similar algorithm is readily available in various solvers such as DICOPT by Grossmann et al. (2002).

To insure that the arc capacity never goes to 0 and the congestion probability does not make the model infeasible, we set up the following bounds for these variables.

$$\gamma \in [0.02, 0.99],$$
$$\mathcal{Q}_a \in [0.3\beta_a, \beta_a].$$

Given the path set and the clearance time T, three sets of experiments are performed for the model. We first present the results of each individual experiment and later a detailed discussion of the results is presented.

Case 1: Objective function is set to minimization of congestion in the network. In this case, there is no constraint on the number of paths being used for evacuation. Model MCP-J is solved for different values of T to find the minimum congestion level that can be achieved. It should be noted that the computation of the capacity approximation is done for different probability levels within the model and the final capacity that is used in the solution is obtained from the minimum probability level found by the objective function. Table 2 shows the result of the experiment.

Table 2: Minimum congestion probability attainable for clearance time T

Clearance Time	Congestion Probability	No. of paths
130	0.466	25
131	0.426	29
132	0.395	27
133	0.381	29
134	0.348	28
135	0.317	24
136	0.289	22
137	0.259	28
138	0.252	23
139	0.229	25
140	0.200	24
141	0.189	27
142	0.183	25
143	0.164	22
144	0.144	30
145	0.140	26
150	0.085	28

Case 2: Objective function is set to minimization of number of evacuation paths. Solution for this case for time $T = 130$ produced the following results.

- Number of paths = 16
- Congestion probability = 0.852

Case 3: Objective function is set to minimization of congestion probability with an added constraint of limiting the total number of paths to be used for evacuation to a value N. Solving the model for different values of N, we obtained the result shown in Table 3.

Table 3: Congestion probability attainable for clearance time T

	Number of Paths (N)				
Clearance Time	20	21	22	23	24
130	0.468	0.468	0.468	0.468	0.467
135	0.322	0.322	0.321	0.321	0.317
140	0.209	0.208	0.208	0.207	0.200
145	0.144	0.144	0.144	0.143	0.141
150	0.091	0.089	0.089	0.087	0.087

After analyzing the results from all the three experiments, it can be concluded that for achieving a congestion level below 10%, the clearance time has to be increased to 150. Even if the number of paths are increased, the congestion level cannot be reduced below a certain level for evacuation being completed within a given time. This is because of presence of bottleneck at certain sections of road which are being shared by multiple paths and cannot be avoided. For these bottleneck locations being shared by multiple paths, the flow has to be adjusted to accommodate more paths sharing the same arc and maintain the free flow with the same level of probability. Results obtained from the experiment in Case 1 provides the best bound achievable for the congestion probability within time T without any restriction on the number of evacuation paths. As seen from results in Table 2, even though there was no restriction on number of paths to be used, the congestion level did not go down after a certain probability level has reached for any particular input time T. This hypothesis is further proved by the experiment described in Case 3. From Table 3, we can see that the model achieved a comparable performance in terms of congestion probability for a given clearance time with a limited number of evacuation paths. This congestion level was achieved even though the number of evacuation paths was limited to N whereas more paths were used to achieve the same level of congestion in Case 1.

From the experiment of Case 2, it was found that a minimum of 16 paths are required for evacuation in clearing time $T = 130$ and this would result in a congestion probability of $\gamma = 0.85$. This congestion probability is quite high to tolerate for any practical evacuation plan and would result in heavy traffic buildup and eventually leading to require more time for evacuation. It should be noted that to achieve a traffic flow with a congestion level below a desired probability within a limited time T, the evacuation planner should consider evacuating less number of vehicles. An alternative plan would be to provide local shelters for people who are left behind.

4 Conclusion

In this paper, we extended the concept of stochastic capacity in the evacuation planning problem and formulated the problem using the notion of congestion minimization in evacuation routes. Traditionally, clearance time estimates and route planning are determined considering a deterministic scenario. Only a handful of literature consider capacity as random variable and design the model to find the clearance time estimates. To capture the variation of capacity, we first formulated a probabilistic constraint for arc capacity violation in the proposed minimum cost network flow model. We assumed that the random capacity follows Weibull distribution and estimated the clearance time based on a desired reliability level. Bottlenecks are a result of traffic flow reaching the saturation point of the capacity. We explicitly considered the uncertainty of traffic jam inherent in high volume traffic that occurs in evacuation. To alleviate this problem, we used the bottleneck minimization principle and developed a model that minimizes the probability of traffic congestion for a given network state.

Numerical experiments showed that assigning traffic flow in anticipation of capacity degradation would result in a conservative plan compared to deterministic models, but such plans are more reliable. Paths and the corresponding flow that would result in minimum congestion were found using the MCP-J model. For a given network and clearance time, a state is reached where increasing the number of evacuation paths will not have any effect in decreasing the congestion probability below a certain level as the bottleneck arc is being shared by multiple paths. By providing the reliability level, stochastic model equips the evacuation planner to make probabilistic inference about the model results.

Finally, we conclude that minimization of clearance time for evacuation plan is not the primary goal that the planner should look for. Since the capacity is variable and there is a metastable region of the arc capacity in which there is a finite probability of congestion to occur, slight disturbance can cause the traffic breakdown and increase the total clearance time altogether. Therefore, an evacuation plan considering variable parameters should be used such that the tolerable level of violations of the probabilistic constraints can be inferred. The results obtained using the stochastic models are more practical considering the dynamic and uncertain nature of events during evacuation.

Future research in the direction of incorporating uncertainty in the evacuation planning model would be to consider the random demand of the number of vehicles. Owing to the non-existence of any distribution function for estimating the number of evacuees, distribution free chance-constrained models would be a better alternative to model the problem and coming up with a robust plan.

Appendices

A Deterministic MCP Model

A fixed-flow model with the objective of minimum number of paths to be used for evacuation can be formulated as follows.

$$\text{Minimize} \quad \sum_{p \in \mathscr{P}} y_p \tag{32}$$

$$\text{Subject to:} \quad \sum_{p \in \mathscr{P}} f_p \cdot \delta_{pa} \leq \mathscr{U}_a, \qquad \forall a \in \mathscr{A}; \tag{33}$$

$$\sum_{p|\mathbb{O}_p = i} (T - \sigma_p) f_p \geq \mathcal{S}_i, \qquad \forall i \in \mathscr{N}_c; \tag{34}$$

$$\sum_{p|\mathbb{D}_p = j} (T - \sigma_p) f_p \leq \mathscr{C}_j, \qquad \forall j \in \mathscr{N}_s; \tag{35}$$

$$(T - \sigma_p) f_p \leq M \cdot y_p, \qquad \forall p \in \mathscr{P}; \tag{36}$$

$$f_p \in \mathbb{Z}^+, y_p \in \{0, 1\} \qquad \forall p \in \mathscr{P}; \tag{37}$$

We define the decision variables $f_p \in \mathbb{Z}^+, y_p \in \{0, 1\}, \forall p \in \mathscr{P}$ for the model. Constraint (33) ensure that the sum of flows for all paths p on any arc $(i, j) \in \mathscr{A}$ should not exceed the maximum capacity of that arc. Constraint (34) guarantees that the sum of flows on path originating from the nodes in $\mathscr{N}_c$ over all time is greater than or equal to the supply at that node. This constraint generates the flow in the paths that are selected for the solution. Constraint (35) ensures that the summation of flow on paths coming into the destination over all time do not exceed the capacity $\mathscr{C}_j$ of the destination nodes $\mathscr{N}_s$. Constraint (36) limits the total flow over all time on any path p if the path is selected in the solution. If the path is not selected than the flow of the path at all times is set to 0. If the path is selected, the summation of the flows is limited to $M = max(\mathcal{S}_{i|\mathbb{O}_p})$, i.e., the maximum possible supply initially present at the origin of any path. Constraint (37) forces the variables f_p and y_p to take integer and binary values respectively. This model gives the flexibility to the emergency managers for finding the minimum numbers of paths that are required for evacuation. Using less paths for evacuation is helpful to emergency managers for efficient management of the evacuation process in wake of limited resources.

References

Aronson, J. (1989), 'A survey of dynamic network flows', *Annals of Operations Research* **20**(1), 1–66.

Ben-Tal, A., Chung, B., Mandala, S. and Yao, T. (2011), 'Robust optimization for emergency logistics planning: Risk mitigation in humanitarian relief supply chains', *Transportation Research Part B: Methodological* **45**(8), 1177–1189.

Brilon, W., Geistefeldt, J. and Regler, M. (2005), Reliability of freeway traffic flow: a stochastic concept of capacity, *in* ‘Proceedings of the 16th International symposium on transportation and traffic theory’, pp. 125–144.

Charnes, A. and Cooper, W. (1959), ‘Chance-constrained programming’, *Management science* **6**(1), 73–79.

Chen, A., Yang, H., Lo, H. and Tang, W. (2002), ‘Capacity reliability of a road network: an assessment methodology and numerical results’, *Transportation Research Part B: Methodological* **36**(3), 225–252.

Chiu, Y., Zheng, H., Villalobos, J. and Gautam, B. (2007), ‘Modeling no-notice mass evacuation using a dynamic traffic flow optimization model’, *IIE Transactions* **39**(1), 83–94.

Choi, W., Hamacher, H. and Tufekci, S. (1988), ‘Modeling of building evacuation problems by network flows with side constraints’, *European Journal of Operational Research* **35**(1), 98–110.

Chung, B., Yao, T., Xie, C. and Thorsen, A. (2011), ‘Robust optimization model for a dynamic network design problem under demand uncertainty’, *Networks and Spatial Economics* **11**(2), 371–389.

Cova, T. and Johnson, J. (2003), ‘A network flow model for lane-based evacuation routing’, *Transportation research part A: Policy and Practice* **37**(7), 579–604.

Grossmann, I., Viswanathan, J., Vecchietti, A., Raman, R. and Kalvelagen, E. (2002), ‘GAMS/DICOPT: A discrete continuous optimization package’, *GAMS Corporation Inc* .

Hamacher, H. and Tjandra, S. (2002), ‘Mathematical modelling of evacuation problems: A state of art’, *Pedestrian and Evacuation Dynamics* **24**(24), 227–266.

Highway Capacity Manual (2000), *Transportation research board, National Research Council, Washington, DC* .

Hoppe, B. and Tardos, É. (1994), Polynomial time algorithms for some evacuation problems, *in* ‘Proceedings of the fifth annual ACM-SIAM symposium on Discrete algorithms’, Society for Industrial and Applied Mathematics, pp. 433–441.

Kerner, B. (2011), ‘Optimum principle for a vehicular traffic network: minimum probability of congestion’, *Journal of Physics A: Mathematical and Theoretical* **44**, 092001.

Kim, S. and Shekhar, S. (2005), Contraflow network reconfiguration for evacuation planning: a summary of results, *in* ‘Proceedings of the 13th annual ACM international workshop on Geographic information systems’, ACM, pp. 250–259.

Lim, G. and Reza Baharnemati, M. (2010), An evacuation planning framework: A path-based optimization approach, *in* ‘Proceedings of IIE Annual Conference’.

Lim, G., Zangeneh, S., Reza Baharnemati, M. and Assavapokee, T. (2012), 'A capacitated network flow optimization approach for short notice evacuation planning', *European Journal of Operational Research* **223**, 234–245.

Litman, T. (2006), 'Lessons from katrina and rita: What major disasters can teach transportation planners', *Journal of Transportation Engineering* **132**(1), 11–18.

Liu, Y., Lai, X. and Chang, G. (2006), 'Cell-based network optimization model for staged evacuation planning under emergencies', *Transportation Research Record: Journal of the Transportation Research Board* **1964**(1), 127–135.

Lo, H. and Tung, Y. (2003), 'Network with degradable links: capacity analysis and design', *Transportation Research Part B: Methodological* **37**(4), 345–363.

Lu, Q., George, B. and Shekhar, S. (2005), 'Capacity constrained routing algorithms for evacuation planning: A summary of results', *Advances in Spatial and Temporal Databases* pp. 291–307.

Mahnke, R., Kaupuzs, J. and Lubashevsky, I. (2005), 'Probabilistic description of traffic flow', *Physics Reports* **408**(1-2), 1–130.

Miller-Hooks, E. and Sorrel, G. (2008), 'Maximal dynamic expected flows problem for emergency evacuation planning', *Transportation Research Record: Journal of the Transportation Research Board* **2089**(1), 26–34.

Nemirovski, A. and Shapiro, A. (2007), 'Convex approximations of chance constrained programs', *SIAM Journal on Optimization* **17**(4), 969–996.

Ng, M. and Waller, S. (2010), 'Reliable evacuation planning via demand inflation and supply deflation', *Transportation Research Part E: Logistics and Transportation Review* **46**(6), 1086–1094.

Ng, M. and Waller, S. (2011), 'A dynamic route choice model considering uncertain capacities', *Computer-Aided Civil and Infrastructure Engineering* **27**(4), 231–243.

Orda, A. and Rom, R. (1990), 'Shortest-path and minimum-delay algorithms in networks with time-dependent edge-length', *Journal of the ACM (JACM)* **37**(3), 607–625.

Persaud, B., Yagar, S. and Brownlee, R. (1998), 'Exploration of the breakdown phenomenon in freeway traffic', *Transportation Research Record: Journal of the Transportation Research Board* **1634**(1), 64–69.

Prékopa, A. (1970), On probabilistic constrained programming, *in* 'Proceedings of the Princeton Symposium on Mathematical Programming', Citeseer, pp. 113–138.

Rungta, M., Lim, G. and Baharnemati, M. (2012), 'Optimal egress time calculation and path generation for large evacuation networks', *Annals of Operations Research* .
URL: *http://dx.doi.org/10.1007/s10479-012-1225-z*

Siu, B. and Lo, H. (2008), 'Doubly uncertain transportation network: Degradable capacity and stochastic demand', *European Journal of Operational Research* **191**(1), 166–181.

Stepanov, A. and Smith, J. (2009), 'Multi-objective evacuation routing in transportation networks', *European Journal of Operational Research* **198**(2), 435–446.

Travis Waller, S. and Ziliaskopoulos, A. (2006), 'A chance-constrained based stochastic dynamic traffic assignment model: Analysis, formulation and solution algorithms', *Transportation Research Part C: Emerging Technologies* **14**(6), 418–427.

Wardrop, J. (1952), Some theoretical aspects of road traffic research, *in* 'Proceedings of the Institution of Civil Engineers', Vol. 1, pp. 325–378.

Yao, T., Mandala, S. and Chung, B. (2009), 'Evacuation transportation planning under uncertainty: a robust optimization approach', *Networks and Spatial Economics* **9**(2), 171–189.

Yazici, A. and Ozbay, K. (2010), 'Evacuation network modeling via dynamic traffic assignment with probabilistic demand and capacity constraints', *Transportation Research Record: Journal of the Transportation Research Board* **2196**, 11–20.

Yusoff, M., Ariffin, J. and Mohamed, A. (2008), Optimization approaches for macroscopic emergency evacuation planning: a survey, *in* 'International Symposium on Information Technology', Vol. 3, IEEE, pp. 1–7.